



NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.
FACULTY OF SCIENCE AND TECHNOLOGY

April/May Examination 2019

COURSE CODE: MTH381

COURSE TITLE: MATHEMATICAL METHODS III

CREDIT UNIT: 3

TIME: 3 HOURS

Total: 70 Marks

INSTRUCTION: Attempt question One (1), and any other four questions.

Question 1

(a) If $f(x, y) = x^2 - 2xy + y^2$

Find (i) $f(1, -1)$ (ii) $f(2, 1)$ [2 marks each]

(b) If $u = x + y + z$, $v = x^3 + y^3 + z^3$ and $w = xyz$

find the Jacobian $J = \frac{\partial(u, v, w)}{\partial(x, y, z)}$ [6 marks]

(c) Determine the Fourier series of the function defined by

$$f(x) = 2x \quad 0 < x < 2\pi$$

$$f(x + 2\pi) = f(x) \quad [6 \text{ marks}]$$

(d) Express the following in polar form stating the modulus of the vector and argument (the principal) of the vector value:

(i) $1 + i$ (ii) $-5 + 5i$ [3 marks each]

Question 2

(a) Determine whether the following pair of functions are linearly dependent as the case may be

(i) $u(x) = x^2$, and $v(x) = 3x^2$ [3 marks]

(ii) $u(x) = \cos 2x$, and $v(x) = \sin 2x$ [3 marks]

(b) State the Residue theorem. [2 marks]

- (c) Evaluate $\oint_C \frac{e^z}{(z+1)^2} dz$ where C is the circle $|z-1|=3$ [4 marks]

Question 3

- (a) Determine the poles of the function

$$\frac{z^3}{(z-1)^3(z+3)}$$

And the residue at each pole. [3 marks]

- (b) Verify divergence theorem for the vector field

$$F = z\mathbf{i} + y^2\mathbf{j} + x\mathbf{k}$$

over the region bounded by the planes

$x=0, x=1, y=0, y=1, z=0$ and $z=1$. [9 marks]

Question 4

(a) If $f(x, y) = \frac{3x+2y}{4-2xy}$

Find (i) $f(0,1)$ (ii) $f(1,3)$ [2 marks each]

- (b) Using Laplace transformation, solve the initial value problem:

$$y'' - 3y' - 2y = 4t; \quad y(0) = 1 \text{ and } y'(0) = -1$$
 [8 marks]

Question 5

- (a) Evaluate the double integral

(i) $\int_{y=1}^{y=2} \int_{x=0}^{x=3} (x^2 + y) dx dy$ (ii) $\int_1^2 \int_1^3 x^2 y dx dy$ [6 marks each]

Question 6

- (a) If $A = (2x^2 + 5y)\mathbf{i} - 10yz\mathbf{j} + 5xz^2\mathbf{k}$ evaluate $\int_C A \cdot dr$ from $(0,0,0)$ to $(1,1,1)$ along the following parts C: $x=t, y=t^2$, and $z=t^3$

The straight lines from $(0,0,0)$ to $(1,0,0)$ then to $(1,1,0)$ and then to $(1,1,1)$.

The straight line joining $(0,0,0)$ and $(1,1,1)$.

- (b) Given that $z_1 = 3 - 4i$ & $z_2 = -6 + i$; find (i) $z_1 z_2$ (ii) $\frac{z_1}{z_2}$ [3 marks each]

End of Examination questions.