



NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, NnamdiAzikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
January\February Examination 2018

Course Code: MTH341
Course Title: REAL ANALYSIS II
Credit Unit: 3
Time Allowed: 3 HOURS
Instruction: ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS

1. (a) State and prove the Lagrange's Mean Value theorem **(3 Marks)**
(b) Verify Rolle 's Theorem for the functions in
(i) $f(x) = (x + 2)^3(x - 3)^4$ in $[-2,3]$ **(5 Marks)**
(ii) $f(x) = 1 - (x - 3)^{\frac{2}{3}}$ in $(2,4)$ **(5 Marks)**
(c) (i) Verify whether or not Rolle's theorem can be verified for $f(x) = \frac{x^2 - 4x}{x + 2}$ lying in $(0,4)$. **(5 Marks)**
(ii) What is the point of discontinuity $f(x) = \frac{x^2 - 4x}{x + 2}$? **(4 Marks)**
2. (a) Separate the intervals in which the polynomial $f(x) = (4 - x^2)^2$ is increasing or decreasing. **(3 Marks)**
(b) (i) Show that, for any $x \geq 0$, $1 + x < e^x < 1 + xe^x$ **(5 Marks)**
(ii) Verify Lagrange's Mean Value theorem for the functions $x(x - 1)(x - 2)$ in $(0, 1/2)$ **(4 Marks)**
3. (a) Verify Cauchy's mean value theorem for the functions $f(x) = \ln x$, $g(x) = \frac{1}{x}$ in $[1,e]$ **(6 Marks)**
(b) Calculate approximately $\sqrt[5]{245}$ by using Lagrange's Mean Value theorem **(6 Marks)**

4. Evaluate the following limits

(i) $\lim (x^2 + 1)(x^3 - 4)$ as x tends to 2

(ii) $\lim \frac{x^2 - 4}{3x - 6}$ as x tends to 2

(iii) $\lim \left(\frac{\cos x - 1}{x} \right)$ as x tends to zero

(iv) $\lim \left(\frac{\sin x}{x} \right)$ as x tends to zero

(3marks each)

5. (a) Find the first 3 terms in the Taylor series for

(i) $\ln x$ at $x=1$

(3 Marks)

(ii) $3x^5 - x^4 + 2x^3 + x^2 - 2$ at $x = -1$

(3 Marks)

(a) Find the first 3 terms in the Maclaurin's series for (i) $\sin 2x$

(2 Marks)

(ii) $\frac{x}{\sqrt{1-x^2}}$

(2 Marks)

(iii) xe^{-x}

(2 Marks)

6. (a) (i) Find the nature of the stationary points for the function $y = 3x^4 - 8x^3 + 6x^2 + 5$ **(4 Marks)**

(ii) Find the maximum and minimum values of $3\sin x + 4\cos x$, and values of x ($0^\circ \leq x \leq 360^\circ$) where they occur **(4 Marks)**

(b) Find the maximum and minimum values of $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

(4 Marks)