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NATIONAL OPEN UNIVERSITY OF NIGERIA Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES April Examination 2019

Course Code: Course Title:	MTH341 Real Analysis II
Credit Unit:	3
Time Allowed:	3 HOURS
Total:	70 Marks
Instruction:	ATTEMPT QUESTION NUMBER ONE AND ANY OTHER FOUR (4) QUESTIONS

- 1 (a) State L-Hospital's rule
 - (b) Suppose that f and g are two functions differentiable at $a \in R$. Prove the Product Rule for differentiation, namely that (fg)'(a) = f(a)g'(a) + (f)'(a)g(a) using the Rules for Limits **(6marks)**
 - (c) Find the value of $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$ (6marks) (d) if $f(x) = e^x + x\cos(x)$ and $g(x) = \sin(x)$ then find the value of $\lim_{x \to \infty} \frac{f(x)}{x}$ (8marks)

(d) if $f(x) = e^x + x\cos(x)$ and $g(x) = \sin(x)$ then find the value of $\lim_{x \to 0} \frac{f(x)}{g(x)}$. (8marks)

2 (a) (i) Define a derivative of a function in an interval.

(ii) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = x^2 \cos(\frac{1}{x})$ if $x \neq 0$ and f(0) = 0. Find the derivative of

$$f at x = 0 if it exists.$$
(4marks)

(b) Let
$$f: R \to R$$
 be defined as $f(x) = \begin{cases} x^4 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ (5marks)

Show that f'(0) exist. And hence find it value.

3 (a) State and proof Rolle's theorem(6marks)(b) (i) if
$$f(x) = x^2 - 3x + 2$$
 and $g(x) = x^3 - x^2 + x - 1$ then find the value of $\lim_{x \to 1} \frac{f(x)}{g(x)}$.(3marks)(ii) Show that $\sqrt{3}$ is irrational(3marks)

- 4 (a) Does there exist a differentiable function $f: \mathfrak{R} \to \mathfrak{R}$ such that f'(0) = 0 but $f'(x) \ge 1$ for $x \ne 0$?
 - (b) Write out the Taylor polynomial $P_2(x)$ of order two at x = 0 for the function g and give an expression for the remainder $R_2(x)$ in Taylor's formula $g(x) = \sqrt{1+x} = P_2(x) + R_2(x) 1 < x < \infty$. (7marks)

(3marks)

(5marks)

(2marks)

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- 5 (a) (i) Let $\{p_n\}$ be a sequence of real numbers. Give the definition of the convergence of this sequence **(6marks)**
 - (ii) What are intermediate forms?

(b) Find (i)
$$\lim_{(x,y)\to(0,0)} \frac{x^3 + 3xy^2 - xy^2}{x^2 + xy}$$
 (ii) $\lim_{(x,y)\to(0,0)} \frac{y^7 x^{98} - x^{97} y^8 + x^{105}}{xy^7 + x^8}$ (6marks)

- 6 (a) Verify the Cauchy's mean value theorem for the functions $f(x) = \sin x$ and $g(x) = \cos x$ in the interval $\left[-\frac{\Pi}{2}, 0\right]$. (6marks)
 - (b) Let the functions f and g be defined by $f(x) = e^x$ and $g(x) = e^{-x}$ for all $x \in [a,b]$. Show that 'c' obtained from the Cauchy's mean value theorem is the arithmetic mean of a and b. (6marks)