NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
January\February Examination 2018

Course Code:	**MTH312**
Course Title:	**Abstract Algebra II**
Credit Unit:	**3**
Time Allowed:	**3 HOURS**
Instruction:	**ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) Given that $n \in N$. Show that the composition $fof : Z \to Z; \quad f(x) = nx$ and $G : Z \to Z/nZ : \; g(x) = x$ is a
   (i) homomorphism
   (ii) What is Ker f(gof)?
   (iii) Find Im(gof) **(2 Marks)**

   (ii) Show that every subgroup H of an abelian group G is a normal subgroup. **(4 Marks)**

   (b) Given H as the subgroup of S_3 consisting of elements (1) and (12) and W consisting of permutations (1), (123), and (132), show that H is not a normal subgroup but W is a normal subgroup. **(8 Marks)**

   (c) Given that N is a subgroup of G. Show that the following statement are equivalent
   i). the subgroup N is normal in G **(2 Marks)**
   ii). for all $g \in G$, $\quad g^{-1}Ng \subset N$ **(3 Marks)**
   iii). For all $g \in G$, $g^{-1}Ng = N$ **(3 Marks)**

2 (a) let $f : G_1 \to G_2$ be a group homomorphism, show that
   (i) Ker f is normal subgroup of G_1 **(4 Marks)**
   (ii) Im f is a subgroup of G_2 **(4 Marks)**

   (b) If $f : G_1 \to G_2$ is an onto group homomorphism and S is a subset that generates G_1, show that f(S) generates G_2 **(4 Marks)**

3 (a) If H and K are subgroups of a group G, with K normal in G, show that $H/(H \cap K) \approx (HK)/K$ **(4 Marks)**

   (b) If a group G be the internal direct product of its subgroups H and K, show that:

   (i) Each $x \in G$ can be uniquely expressed as x=hk, where $h \in H, k \in K$ **(4 Marks)**
   (ii) hk=kh $\quad \forall h \in H, k \in K$ **(4 Marks)**

4 (a) Let R be a Boolean ring (i.e. $a^2 = a \forall a \in R$), show that $a = -a \forall a \in R$, also show that R must be commutative **(3 Marks)**

(b) Let R be a ring, if for a,b,c elements of R, show that

(i) a0 = 0 = 0a **(3 Marks)**

(ii) a(-b) = (-a)b = -(ab) **(3 Marks)**

(iii) (-a)(-b) = ab **(3 Marks)**

5 (a) (i) Find the principal ideals of Z_{10} generated by $\bar{3}$ and $\bar{5}$ **(3 Marks)**

(ii) Find the nil radicals of Z_8 and p(X) **(3 Marks)**

(b) Let R be a ring with identity 1, if 1 is an ideal of R and $1 \in 1$, show that 1=R **(6 Marks)**

6. a) State and prove the Cayley's theorem. **(6 Marks)**

b) Use the Cayley's table to show that $(Z_5, +)$ is a group **(6 Marks)**