NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
January\February Examination 2018

CODE:MTH 311 **TIME:** 3 HOURS
TITLE: CALCULUS OF SEVERAL VARIABLES **CREDIT UNIT: 3**
TOTAL: 70 MARKS
INSTRUCTION: QUESTION ONE (1) IS COMPULSORY AND ATTEMPT ANY OTHER 4

1a) Define the following;

i) A real-valued function of two variables?

ii) Partial derivative of a function of two or more variables with respect to one of its variables

iii) Total derivative of the function $(x, y, z, \dots, u)$. Hence evaluate the total derivatives of $F(x, y, z) = 4x^2y^3 + z^2$ and $F(x, y, z) = 2x^2y^3 - 3z^2$

b) Let f be a function defined by $f(x, y) = (x^2 + y, xy)$.
Find i) $f(2,3)$ ii) $f(3,2)$ iii) $f(-2,-3)$

c) State the Clairaut's Theorem. Verify the theorem with $F(x, y) = y^2e^{2x} + \cos 4y$ **18marks**

2) Let f, g and h be functions defined by $f(x) = 7x - 3$, $g(x) = x + 2$ and $h(x) = 3x^2 - 7x - 5$. Find:

i) $h(x-2)$ ii) $f\big(g(x)\big)$ iii) $g\big(f(x)\big)$ iv) $(f+g)(x)$ v) $h\Big(g\big(f(x)\big)\Big)$ **13marks**

3) Let $x = r\cos\theta$ and $y = r\sin\theta$. What is the Jacobian determinant (r, θ)? **5marks**
Obtain the Jacobian determinant such that
$y_1 = 5x_2$; $y_2 = 4x_1 - 2\sin(x_2x_3)$ and $y_3 = x_2x_3$ **8marks**

4a) Find $f_{xx}, f_{xy}, f_{yx}, f_{yy}$ of the following:

i) $f = 2x^3 - xy^2 - y^4$ **3marks**

ii) $f = 3e^{-xy} - y\cos x$ **3marks**

b) Let $z = e^{\cos x^2}$. Solve $\frac{dz}{dx}$ by the chain rule. **7marks**

5) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for each of the following implicit functions:

i) $z^2 - 2x^4yz^3 = 3x^3 - y^2$ **6 marks**

ii) $y\cos(4xz) = 2z^3 - x^2\, sin(2xy)$ **7marks**

6 a) Compute a second order Taylor Series expansion around the origin of the function

$f(x, y) = e^x \log(1 + y)$ **3.5marks**

b) State the i) necessary ii) sufficient conditions for a maxima or minima of the function:

$z = f(x, y)$. **5marks**

c) Hence find the maxima and minima of the function $z = 2x^2 + xy - y^2 + y$ **4.5marks**