**NATIONAL OPEN UNIVERSITY OF NIGERIA**
**University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja**

**FACULTY OF SCIENCES**
**April/May 2019 Examinations**

**Course Code:** MTH311
**Course Title:** Calculus of Several Variables
**Credit Unit:** 3
**Time Allowed:** 3 Hours
**Total:** 70 Marks
**Instruction:** Answer Question One and Any Other 4 Questions

1a) Let $f$ be a function defined by $f(x, y) = (x^2 + y, xy)$

Find   i) $f(2, -3)$   ii) $f(3,2)$   iii) $f(-2, 3)$   iv) $f(-2, -3)$     **(8 marks)**

b) Evaluate i) $\lim_{\substack{x \to 1 \\ y \to 2}} \dfrac{3x^2 y}{x^2 + y^2 + 5}$     **(3 ½ marks)**

ii) $\lim_{\substack{x \to \infty \\ y \to 3}} \dfrac{2x - 3}{x^3 + 4y^3}$     **(3 ½ marks)**

c) If $u = (1 - 2xy + y^2)^{-1/2}$ prove that    $x\dfrac{\partial u}{\partial x} - y\dfrac{\partial u}{\partial y} = y^2 u^3$     **(7 marks)**

2 a) If $x = r\cos\theta$ and $y = r\sin\theta$, then evaluate $\dfrac{\partial(x,y)}{\partial(r,\theta)}$     **(6 marks)**

 b) i) State the Clairaut's Theorem     **(2 marks)**

   ii) Hence, verify the theorem with $F(x, y) = y^2 e^{2x} + \cos 2y$     **(4 marks)**

3) a) Define total derivative of the function $F(x, y, z, \dots, u)$     **(3 marks)**

 b) Find the value of the Jacobian $\dfrac{\partial(u,v)}{\partial(r,\theta)}$, where $u = x^2 - y^2$, $v = 2x$ and

   $x = r\cos\theta$, $y = r\sin\theta$.     **(9 marks)**

4a) Find $f_{xx}, f_{xy}, f_{yx}, f_{yy}$ of the following:

i)      $f = 2x^3 - xy^2 - y^4$                                     **3marks**

ii)      $f = 3e^{-xy} - y \cos x$                                 **3marks**

b) Let $z = f(x, y)$ where $x = e^u \cos v, \quad y = e^u \sin v$ show that

$$y \frac{\partial z}{\partial u} + x \frac{\partial z}{\partial v} = e^{2u} \frac{\partial z}{\partial y}$$               **6marks**

5) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for each of the following implicit functions:

i)      $z^2 - 2x^4yz^3 = 3x^3 - y^2$                       **6 marks**

ii)      $y \cos(4xz) = 2z^3 - x^2 \sin(2xy)$             **6 marks**

6 a) Compute a second order Taylor Series expansion around the origin of the function

         $f(x, y) = e^x \log(1 + y)$                  **3 marks**

(b) State the (i) necessary and (ii) sufficient conditions for a maxima or minima of the function $z = f(x, y)$

                                            **5marks**

c) Hence find the maxima and minima of the function $z = 2x^2 + xy - y^2 + y$

                                            **4 marks**