



NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
January\February Examination 2018

Course Code: MTH309
Course Title: Optimization Theory
Credit Unit: 3
Time Allowed: 3 HOURS
Instruction: ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS

1. (a) Explain the following terms in the context of operations research
- (i) Network scheduling
 - (ii) Game theory
 - (iii) Decision theory
 - (iv) Queuing theory
- (4 Marks)**
- (b) A firm manufactures 3 products A , B and C , the profits are ~~₦3~~, ~~₦2~~ and ~~₦3~~ respectively. The firm has 2 machines and given below is the required processing time in minutes for each machine on each product.

	Product A	Product B	Product C
Machine M_1	4	3	5
Machine M_2	3	2	4

Machine M_1 and M_2 have 2000 and 2500 machines minutes respectively. The firm must manufacture 100 A 's, 200 B 's and 50 C 's but not more than 150 A 's. Set up a linear programming problem to maximize the profit.

(8 Marks)

- (c) What is linear programming? **(2 Marks)**
- (d) A paper mill produces two grades of paper namely X and Y . Due to raw materials restrictions, it cannot produce more than 400 Tonnes of grade X and 300 Tonnes of grade Y in a week. There are 160 production hours in a week. It requires 0.2 and 0.4 hours to produce a Tonne of products X and Y respectively with the corresponding profits of ₦200 and ₦500 per tonne. Formulate the above as a linear programming problem to maximize profit and find the optimum product mix. **(8 Marks)**

2. (a) A company manufactures two products A and B . Each unit of B takes twice as long produce as

one unit of A and if the company was to produce only A , it would have time to produce 2000 units per day. The availability of the raw material is sufficient to produce 1500 units per day of both A and B combined. Product B requires a special ingredient; only 600 units can be made per day. If A fetches a profit of ₦2 per unit and B a profit of ₦4 per unit. Find the optimum product mix. (6 Marks)

- (b) Consider the following linear programming problem.

$$\text{Maximize } z = 5x_1 + 4x_2$$

Subject to :

$$x_1 - x_2 \leq 8$$

$$x_1 \leq 7$$

(6 Marks)

$$x_1, x_2 \geq 0.$$

3. (a) Use sensitivity analysis simplex method to solve the following linear programming problem.

$$\text{Maximize } z = 20x_1 + 10x_2$$

Subject to :

$$x_1 + x_2 \leq 3$$

$$3x_1 + x_2 \leq 7 \quad (4 \text{ Marks})$$

$$x_1, x_2 \geq 0.$$

- (b) A manufacturer produces three types of plastic fixtures. The time required for molding, trimming and packaging is given as follows:

Process	Type A	Type B	Type C	Total time available
Molding	1	2	$\frac{3}{2}$	12000
Trimming	$\frac{2}{3}$	$\frac{2}{3}$	1	4600
Packaging	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{2}$	2400
Profit	₦11	₦16	₦15	-

How many dozen of each type of fixture should be produced to obtain a maximum profit?

(8 Marks)

4. (a) Babies are born in a sparsely populated state at the rate of one birth every 12 minutes. The time between births follows an exponential distribution. Find the following:
- The average number of births per year
 - The probability that no birth will occur in any one day
 - The probability of issuing 50 birth certificates in 3 hours given that 40 certificates were issued the first 2 hours of the 3-hour period. (6 Marks)
- (b) List the phases or stages of operations research. (3 Marks)
- (c) Mention five areas where operations research can be applied. (3 Marks)
5. A boy who sells newspaper, buys newspaper at 0.45 kobo each and sells them for 0.75 kobo each. The condition here is that the boy cannot return the unsold newspapers. The following table shows

the distribution for the daily demand. If each day demand is independent of the previous day demand.

- (a) Find the cumulative probability distribution **(4 Marks)**
 (b) What is the value of x if the distribution is a valid probability mass function? **(2 Marks)**
 (c) Hence or otherwise, show that the distribution is a valid probability mass function. **(6 Marks)**

Number of Customers	240	250	260	270	280	290	300	310	320	330
Probability	0.01	0.03	0.06	0.10	0.20	0.25	0.15	0.10	0.05	x

6. (a) Solve the game with the pay - off matrix. **(4 Marks)**

		Player B strategies				
Player A strategies		I	II	III	IV	V
	1	-2	5	-3	6	7
	2	4	6	8	-1	6
	3	8	2	3	5	4
	4	15	14	18	12	20

- (b) According to the National Health and Nutrition Examination Survey USA. The prevalence of myopia varies by racial groups as follows. **(4 Marks)**

Racial Group	Percentage Having Myopia
White	26.4
Black	14.5
Hispanic	18.3
Other	20.7
All	23.9

Assume that one person is selected at random from each of the four racial groups .Find the probability that:

- (i) All the first four groups have myopia.
 (ii) None of them has myopia.
 (iii) Now assume that people are chosen at random from the whole population. Find the probability that, if 3 people are chosen, at least 1 has myopia. **(4 Marks)**