NATIONA PEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Z , NnamdiAzikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
January\February Examination 2018

Course Code: **MTH307**
Course Title: **Numerical Analysis II**
Credit Unit: **3**
Time Allowed: **3 HOURS**
Instruction: **ATTEMPTNUMBER ONE (1) AND ANY OTHERFOUR (4) QUESTIONS**

1. (a) Find the best value of a and b so that $y = a + bx$ fits the data given in the table below:

x	0	1	2	3	4
y	1.0	2.9	4.8	6.7	8.6

[3 Marks]

(b). (i) Show that $2\left(T_n(x)\right)^2 = T_{2n}(x) + 1$

where $T_n(x)$ is Chebyshev polynomial. **[4 Marks]**

(ii)Let $f(x) = \sqrt{x+1}$ $(0,1),(3,2),(8,3)$, construct a free cubic spline. **[5 Marks]**

(c) (i)Evaluate $\int_0^{\pi} \sin x dx$, for $n = 6$ by applying Newton- Cotes formula. **[5 Marks]**

(ii) Solve the boundary value problem $y'' + 3y = 0$ $y(0) = 7,\ y(2\pi) = 0$ **[5 Marks]**

2. (a) In the following table some observed values of *x and y* are given

x	2	3	4	5	6	7
y	4	5	5.71	6.25	6.67	7

the law connecting *x and y* is given as: $xy = ax + by$. Find the best value of *a and b.*

[6 Marks]

(b) Find the cubic spline in the table below:

X	0	2	4	6
Y	1	9	41	41

[6 Marks]

3. (a) Find linear and quadratic least square approximation to

$f(x) = e^x$ Using Legendre polynomials. **[6 Marks]**

(b) Use Hermite cubic interpretation to estimate the value of $\sqrt{55}$ taking [6 Marks]

$f(x) = \sqrt{x},\ x_1 = 4,\ x_2 = 16$

4. (a) Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre polynomials. **[6 Marks]**

(b) For points (0,0), (1,0.5), (2,2) and (3,1.5), find the interpolation cubic spline *S(x)* satisfying $S'(0) = 0.2$ and $S'(3) = -1$. **[6 Marks]**

5. (a) Evaluate the integral

$\int_2^3 (4u^2 + 6)du,\quad n = 4$ using trapezium Rule **[6 Marks]**

(b) Evaluate the Integral $\int_0^3 \frac{1}{1+x^5},\ n = 6$, using Simpsons rule

[6 Marks]

6. (a) Find all the solution of the following boundary value problem

$y'' = \lambda y,\quad y(0) = 0,\quad y(\lambda) = 0$ **[6 Marks]**

(b) Solve the Laplace equation

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \text{.....(1)}$$

Subject to the boundary conditions

$v(x,0) = 1,\ v(0,y) = 1,\ v(1,y) = 1,\ v(x,1) = 1$

$0 \le x \le 1,\quad 0 \le y \le 1$

[6 Marks]