

NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, NnamdiAzikiwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
January\February Examination 2018

Course Code: MTH305

Course Title: Complex Analysis II

Credit Unit: 3

Time Allowed: 3Hours

Total Marks: 70%

**INSTRUCTION: ANSWER QUESTION ONE(1) AND ANY FOUR (4)
QUESTIONS (TOTAL = 5 QUESTIONS IN ALL)**

1(a) Expand $f(z) = \cos z$ by taylor series about the point $z = \frac{\pi}{4}$ (8marks)

1(b) Show that the following function are harmonic

(i) $U = 2x(1 - y)$ (3marks)

(ii) $U(x, y) = y^3 - 3x^2y$ (3marks)

1(c) Given that

$z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$
Show that (3marks)

$$z_1 \cdot z_2 = x_1x_2 - y_1y_2 + i(x_1y_2 + x_2y_1)$$

1(d) Express $\cos 3\theta$ and $\sin 3\theta$ in terms of $\sin \theta$ and $\cos \theta$ only and state the real and imaginary parts. (5marks)

Total Marks = 22

(2a) Show that $\cosh^2 z - \sinh^2 z$ is an hyperbolic function

(6marks)

(2b) Find the cube root of 8 in terms of complex number

(6marks)

(3a) Verify that the real and imaginary parts of the function $f(z) = z^2 + 5iz + 3 = i$ satisfy Cauchy-Riemann equation and deduce the analyticity of the function

(8marks)

(3b) Prove that $\cos^2 z + \sin^2 z = 1$

(4marks)

(4a) Solve: $x^2 + (2x + y)i = 3 - 4i$

(6marks)

(4b) List six types of function

(6marks)

(5a) Given that $z = r(\cos\phi + i\sin\phi)$ (i.e in polar form) Find the modulus of z and the principal argument of z.

(5b) Find also $\arg z$, if $z = 2 - 3i$

(6a) Given that $15\left(\cos\frac{15\pi}{4} + i\sin\frac{15\pi}{4}\right) = a + ib$. Find a and b

(6marks)

(6 b) If $Z = (1 - i)(2 + 3i)$ Find:

(i) $\text{Arg}Z = \tan^{-1}(y/x)$

(3marks)

(ii) $\text{Mod}Z = \sqrt{x^2 + y^2}$

(3marks)