NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
January\February Examination 2018

Course Code: MTH304
Course Title: Complex Analysis I
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Iinstruction: Answer Question one and Any other 4 Questions

1. (a) If $z_1 = 2 + i$ and $z_2 = 3 - 2i$, evaluate each of the following.

   (i) $z_1^3 - 3z_1^2 + 4z_1 - 8$ **(4 marks)**

   (ii) $\left|\frac{2z_2+z_1-5-i}{2z_1-z_2+3-i}\right|^2$ **(4 marks)**

   (b) Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, prove that

   (i) $\overline{z_{1+}z_2} = \bar{z}_1 + \bar{z}_2$ **(4 marks)**

   (ii) $|z_1 z_2|$ **(4 marks)**

   (c) (i) Solve the quadratic equation $az^2 + bz + c$ **(2 marks)**

   (Hint: Use complete the square method)

   (iii) Use the solution c(i) to solve the equation $z^2 + (2i - 3)z + 5 - i = 0$ **(4 marks)**

2. (a) Let $w = f(z) = z^2$. Find the of w which correspond to $z = -2 + i$ **(2 marks)**

   (b) Prove that $\sin^2 z + \cos^2 z = 1$ **(6 marks)**

   (c) Evaluate $\lim_{z \to -2i} \frac{(2z+3)(z-1)}{z^2-2z+4}$ using theorems on limits **(4 marks)**

3. Using the definition of first principle, find the derivative of the followings at the point $z = z_0$

   (i) $f(z) = z^3 - 2z$ **(6 marks)**

   (ii) $f(z) = \frac{1+z}{1-z}$ **(6 marks)**

4. (a) Show that the complex function $f(z) = z^3$ satisfy harmonic function **(6marks)**

(b) Let $z = x + iy$, find the real and imaginary parts of the following complex functions

(i) $f(z) = z^2$ (ii) $f(z) = \frac{1}{z}$ (for $z \neq 0$) **(6 marks)**

(5) Use the Cauchy –Riemann equation to show that the following functions are differentiable at any $z \neq 0$

(i) $f(z) = \bar{z}$ **(6 marks)**

(ii) $f(z) = zRe(z)$ **(6 marks)**

6. (a) Expand $f(z) = In\ (1+z)$ in a Taylor series about $z = 0$ **(6 marks)**

(b) If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, prove that

$z_1 z_2 = r_1 r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$ **(6 marks)**