NATIONAL OPEN UNIVERSITY OF NIGERIAN
Plot 91, Cadastral Zone, Nnamdi Azikiwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
JANUARY/FEBRUARY 2017_2 Examination

Course Code: MTH301
Course Title: Functional Analysis I
Credit Unit: 3
Time Allowed: 3Hours
Total Marks: 70%

INSTRUCTION: ANSWER QUESTION ONE(1) AND ANY FOUR (4) QUESTIONS (TOTAL = 5 QUESTIONS IN ALL)

1(a) Define a metric space? Give one example of a metric space. (4½marks

1(b) What is meant by a topological space? Give an example of a topological space. (5marks)

1(c) Define the length or norm of a vector $x \varepsilon R^3$ (8marks)

1(d) Define the length or norm of a vector $x \varepsilon R^2$ (4½marks)
(TOTAL = 22Marks)

2(a) Let $E\|.\|$ be a normed space. Then the addition, scalar multiplication and the norm mapping itself are continuous (with respect to metric induced by the norm). That is, if $X_n \rightarrow x$, $Y_n \rightarrow y$ and in E and $\lambda n \rightarrow \lambda$ in **R.**
Show that $X_n + Y_n \rightarrow x + y$, $X_n \rightarrow x$ and $\lambda_n X_n \rightarrow \lambda x$ and $\| X_n \| \rightarrow \| x \|$ (5½marks)

2(b) Let X be a complete metric space and $\{A_n\}$ is countable collection of dense open subset of X. Show that $\cup A_n$ is not empty. (6½marks)
(TOTAL = 12Marks)

3(a) Let M = { A, d} be a metric space. Given any four points x, y, z, t $\in$ A. Prove that $d(x, z) + d(y, t) \geq |d(x, y) - d(z, t)|$ (6½marks)

3(b) Define a complete normed linear space. Prove that E is complete if and only if every absolutely convergent series in E converges to a part in E

(5½marks)

(TOTAL = 12Marks)

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4(a) Let X be a normed space and let Y be a closed subspace of X. If f is continuous, bounded, real-valued function in Y. Show that we can find a continuous, bounded real-valued function F on X such that F= f and $\|F\|_\infty = \|f\|_\infty$

(5½marks)

4(b) Prove that for any s, t $\varepsilon\Re$,

max(s,v) = ½[t+ v + |t-v|],

min(t,v) = ½[t+v- |t-v|].

(6½marks)

(TOTAL = 12Marks)

5 Let (X, H) and (Y, S) be topological spaces. Show that a function f : X→ Y is continuous is and only if $f(X_a) \rightarrow f(x)$ for every net $(X_a)_{a\varepsilon A}$ such that $X_a \rightarrow x$

(12marks)

6 Let (S, d) and (T, d) be metric spaces and f a mapping of S into T. Let τ_1 and τ_2 be the topologies determined by d and d_1 respectively. Then $f(S, \tau) \rightarrow (T, \tau)$ is continuous if and only if $S_n \rightarrow S \rightarrow f(S_n,\tau) \rightarrow f(s)$; that is if $s_1, s_2, \ldots, s_n, \ldots$, is a sequence of points in (S, d) converging to x, show that the sequence of points $f(s_1), f(s_2), \ldots, f(s_n), \ldots$ in (T, d) converges to f(x). **(12marks)**