NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 EXAMINATION

Course Code: MTH421
Course Title: Ordinary Differential Equations
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question Number One and Any Other Four Questions

1. (a) Define the following .
    i.   Connected Set .   **(2 marks)**
    ii.   Open Set   **(2 marks)**
    iii. Boundary Point   **(2 marks)**
    iv   Convergence of a sequence of real numbers $\{x_n\}$ **(2 marks)**
    v.   Solution to an ODE   **(2 marks)**

   (b) . (i) Find the general solution to the ODE
$$3y'' + 2y' - 2y = 0.$$
   (4 marks)

   (ii) Solve the 1st order ODE
$$\frac{dy}{dx} + y\tan x = \sec x.$$
   (4 marks)

   (c) Obtain the general solution to the equation:
$$(x^2 + 1)\frac{dy}{dx} + (y^2 + 1) = 0.$$
   (4 marks)

2. (a) Find the particular solution to the ordinary differential equation:
$$x\frac{dy}{dx} = y + x, \quad y(0) = 0.$$
   (4 marks)

   (b) Solve the ODE: $y' = -7y$ **(4 marks)**
   (c) Verify if the ODE $(1 + 2xy^3)dx + 3x^2y^2dy = 0$ is exact. **(4 marks)**

3. (a) Solve the ODE:
    $$\frac{2x}{y} y' = \frac{y^2}{x^2}.$$

    (6 marks)

    (b) Solve the exact ODE:
    $$\frac{1}{3}x^3 y^3 dx + \frac{1}{4}x^4 y^2 dy = 0.$$
    .

    (6 marks)

4. (a) Show that the general solution to the ODE:
    $$-\frac{1}{x}\cos xy \, dx - \frac{1}{y}\cos xy \, dy = 0.$$
    satisfies the equation:
    $$f(x) = c + \frac{\sin xy}{xy}, \quad c \text{ constant}.$$
    .

    (6 marks)

    (b) Solve the ODE:
    $$\frac{1}{2}x^2 y^2 dx + \frac{1}{3}x^3 y dy = 0.$$

    (6 marks)

5. (a) Define Exact ODE   **(2 marks)**
    (b) Solve the IVP: $y' + y \tan x = \sin 2x, \quad y(0) = 1.$ **(5 marks)**

    (c) Solve the IVP: $y' - y = e^{2x}.$ **(5 marks)**

6. (b) Find a general solution to the following system of ODEs:
    $$y_1' = 3y_2,$$
    $$y_2' = 12y_1.$$

    (4 marks)

    (a) Solve the following system:
    $$y_1' = 9y_1 + 13.5y_2,$$
    $$y_2' = 1.5y_1 + 9y_2.$$

    (8 marks)