NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 Examinations

Course Code: **MTH402**
Course Title: **General Topology II**
Credit Unit: **3**
Time Allowed: **3 Hours**
Total: **70 Marks**
Instruction: Answer Question One (1) and Any Other 4 Questions

1. (a) Let X be a set. Define a basis for a topology on X. **(6 marks)**

(b) What conditions make a collection C of subsets of a set X, a basis for a topology on X? **(3 marks)**

(c) Let X be a set. Let a topology on X be a collection τ of subsets of X. Show that arbitrary unions $\bigcup_{i=1}^{n} U_i$ of elements of τ are in τ. **(6 marks)**

(d) Show that if **B** is a basis for the topology on X and **C** is the basis for the topology on Y, then the collection D = {B × C : B ∈ **B** and C ∈ **C**} is a basis for the topology on X × Y. **(7 marks)**

2. (a) Define a metric on a set X. **(3 marks)**

(b) Let X and Y be two topological spaces. Let **B** be the collection of all sets of the form U × V, where U is an open subset of X and V is an open subset of Y. i.e., **B:** = {U × V: U is open in X and V is open in Y}.
Show that **B** is basis for topology on X × Y. **(3 marks)**

(c) Prove that the collection

S = $\{\pi_1^{-1}(U): U \text{ is open in } X\} \cup \{\pi_2^{-1}(V): V \text{ is open in } Y\}$

is a subbasis for the product on X x Y. **(6 marks)**

3. (a) Let A be a subset of the topological space X. Let A^0 be the set of all limit points of A and $\bar{A}$ be the closure of A. Show that $\bar{A} = A \cup A^0$. **(5 marks)**

(b) Show that if X is a Hausdorff space, then for all x ∈ X, the singleton set {x}is closed **(7 marks)**

4. (a) Define the following terms:
(i) basis for a topology on a set X. **(3 marks)**
(ii) topology generated by a basis. **(3 marks)**
(b) LET $f: \to y$ the topology on the range Y is given by a basis **B**, show that f is continuous if and only if any basis element B∈ ***B***, the set f^{-1} (B) is open in X. **(6 marks)**

5. (a) State whether each of the following is Hausdorff space or not:
(i) Every metric topology. **(1 mark)**

(ii) Every discrete space. **(1 mark)**
(iii) The real line R with the finite complement topology. **(1 mark)**
(iv) R with the finite complement topology. **(1 mark)**

(b) Show that the Union of a collection of connected subspaces of X that have one point in common is connected. **(8 marks)**

6. (a) Show that the real line R endowed with the standard topology is not compact. **(4 marks)**
(b) State the tube lemma. **(3 marks)**
(c) Let X, Y and Z be topological spaces. If f : X → Y and g : Y → Z are continuous,
show that the map g ∘ f : X → Z is continuous. **(5 marks)**