NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 Examinations

Course Code: MTH401
Course Title: General Topology
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 4 Questions

1a) Define the following terms:
i) An open set in a metric space, **(2 marks)**
ii) An interior point in a metric space, **(2 marks)**
iii) A closed set in a metric space. **(2 marks)**
b) State without prove the Cauchy schwartz's inequality. **(4 marks)**
c) Show that $K = (0,2) \cap [3,4]$ is a subspace of a metric space (E, d) is disconnected. **(6 marks)**
d) Show that $\mathbb{R}^2$ is connected. **(6 marks)**

2a) Define a metric on a nonempty set E. **(2 marks)**
b) State without prove the Minkowski's inequality. **(4 marks)**
c) Verify that $d_\infty(x,y) = \max\limits_{1 \le i \le n}\{|x_i - y_i|\}$ is a metric on $\mathbb{R}^2$. **(6 marks)**

3a)Define the following terms:i.an open ball centred at x_0 of radius $r > 0$. Ii. A sphere centred at x_0 of radius $r > 0$.iii. a close ball centred at x_0 of radius $r > 0$.**(6 marks)**
b) Show that in any metric space (E, d), each open ball is an open set E.**(6 marks)**

4a) Define the following terms:
i) a limit of a sequence $\{x_n\}_{n=1}^{\infty}$ **(2 marks)**
ii) a Cauchy sequence **(2 marks)**
iii) a subsequence of a sequence $\{x_n\}$ **(2 marks)**
b) Given that $\{x_n\} = \{x_n^{(1)}, x_n^{(2)}\}$ is a sequence in $E = (E_1, d_1) \times (E_2, d_2)$. Show that the following are equivalent :
i) $\{x_n\}$ converges in E with respect to the metric ρ_{max}. **(2 marks)**
ii) $\{x_n\}$ converges in E with respect to the metric ρ_2. **(2 marks)**

iii) $\{x_n\}$ converges in E with respect to the metric ρ_1. iv) $\{x_n^{(1)}\}$ and $\{x_n^{(2)}\}$ converges in $(E_1, d_1)$and (E_2, d_2) respectively. **(2 marks)**

5a) State without prove the Pasting Lemma on the union of closed sets. **(4 marks)**

bi) Show that (E, d), if is a metric space, $a \in E$(a fixed element) and $f: E \longrightarrow \mathbb{R}$ such that $f(x) = d(x, a)$ for all $x \in E$. Then f is uniformly continuous on E. **(4 marks)**

bii) Given that $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ such that

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2 + y^2}, & \text{for } x^2 + y^2 \neq 0 \\ 0, & \text{for } x = y = 0 \end{cases}$$

Verify the continuity of f at $(0,0)$. **(4 marks)**

6a) Show that every compact subset of a metric is closed and bounded. **(6 marks)**

b)Show that every real-valued continuous function defined on a compact set is uniformly continuous.

(6 marks)