



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 Examinations

Course Code: MTH 381

Course Title: MATHEMATICAL METHODS III

Credit Unit: 3

Time Allowed: 3 Hours

Total: 70 Marks

Instruction: Answer Question One (1) and Any Other 4 Questions

Q1(a) Define a stationary steady- state vector field. **(2 marks)**

(b) What is the relationship between vector field and vector functions? **(4 marks)**

(c) Find $\int_0^2 \int_0^1 (x^2 + y^2) dy dx$ **(6 marks)**

(d) State the Cauchy's Integral theorem. **(2 marks)**

(e) (i) Evaluate $\int_0^{1+i} z^2 dz$ **(3 mark)**

(ii) Find the residue at the second order pole of $f(z) = \frac{50z}{(z+4)(z-2)^2}$ **(5 marks)**

Q2 (a) Suppose $f(x, y) = x^2 - 4xy + 8y$, find $f(2,3)$ **(4 marks)**

(b) Evaluate $\int_{-2}^2 \int_0^z \int_{x-z}^{x+z} (x + y + z) dy dx dz$ **(8 marks)**

Q3 (a) Define each of the following:

i) A scalar function **(2 marks)**

ii) a differentiable vector function **(2 marks)**

(b) If $A = (3x^2 + 6y)i - 14yzj + 20xz^2k$, evaluate $\int_C A \cdot dr$ from (0,0,0) to (1,1,1).

(8 marks)

Q4, (a) Define a function of two variables. **(2 marks)**

(b) Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ of $u = x^2 + y^2, v = 2xy$. **(5marks)**

c) Show that $\oint_c \frac{dz}{z} = 2\pi i$ **(5 marks)**

Q5. a) Define whether $v(x) = \cos bx$ and $u(x) = \sin bx$ with $b \neq 0$ are linearly dependent or independent. **(5 marks)**

(b) Show that $f(z) = z^3$ satisfies the Cauchy- Riemann equations? **(7 marks)**

Q6(a) Define each of the following:

(i) derivative of a complex function **(3 mark)**

(ii) a differentiable complex function at a point **(3 marks)**

(b) if $z_1 = 3 - 4i$ and $z_2 = 5 + 2i$. Find $\frac{z_1}{z_2}$ **(6 marks)**