



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
**University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja**

**FACULTY OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**  
**2021\_2 Examinations.**

**Course Code:** MTH315  
**Course Title:** Analytical Dynamics  
**Credit Unit:** 3  
**Time Allowed:** 3 Hours  
**Total:** 70 Marks  
**Instruction:** Answer Question One (1) and Any Other 4 Questions

1. (a) Determine the number of degrees of freedom in each of the following cases:
  - (i) 20 particles moving freely in a plane **(2 marks)**
  - (ii) 15 particles moving freely in space . **(2 marks)**

(b) A system of particles consists of a 3-gram mass located at (1, 2, -1), a 5-gram mass at (0,1,3) and 2-gram mass at (1, -1, 1). Find the center of mass. **(6 marks)**

(c) A uniform beam is 10m long and has a mass 10kg and masses of 6kg and 8 kg are suspended from its ends; at what point must the beam be supported so that it may rest horizontally? **(8marks)**

(d) A quadrilateral ABCD has masses 2,3, 5 and 7 units located at its vertices, A(3,-2,2), B(2,-2,3), C(1,-2,4) and D(4,1,3). Find the coordinates of the center of mass. **(4 marks)**
2. Three particles of masses 1,2,3 respectively have position vectors
$$r_1 = (t^2 + 4)i - t^2j + tk,$$
$$r_2 = -2ti + 3t^2j - 2tk,$$
$$r_3 = -t^2i - t^2j + 2tk,$$
where  $t$  is time.  
Find (a) the velocity of the center of mass at time  $t = 0$  **(6 marks)**  
(b) the acceleration at  $t=1$ . **(6 marks)**
3. A particle of mass 12 units moves along a space curve whose position vector is given as a function of time  $t$  by  $r = (t^4 - 3t)i + 6t^3j + t^3k$   
At time  $t = 1$ , find the (a) momentum **(6 marks)**  
(b) force field. **(6 marks)**

4. A particle of mass 3 moves in a force field depending on time  $t$  given by  $F = 9t^2i - 3tj + 6tk$ . Assuming that at  $t = 0$  the particle is located at  $r_0 = -i - j + 2k$  and has velocity  $v_0 = 3i + j - k$ ,  
find (a) the velocity **(6 marks)**  
(b) the position at any time  $t$ . **(6 marks)**
5. A particle moves along the  $x$  axis in a force field having potential  $V = \frac{\alpha}{3}x^3 - \frac{\beta}{2}x^2$ , where  $\alpha$  and  $\beta$  are positive constants. Determine the point(s) of equilibrium. **(12 marks)**
6. (a) State without proof the Liouville's theorem in Hamiltonian theory. **(4 marks)**  
(b) Minimise the integral  $I = \int_0^{\frac{\pi}{2}} \left[ 2 \left( \frac{dy}{dt} \right)^2 - 2y^2 + 4ty \right] dt$ ,  $y(0) = 0$  and  $y\left(\frac{\pi}{2}\right) = 0$  **(8 marks)**