NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 Examinations

Course Code: **MTH311**
Course Title: **CALCULUS OF SEVERAL VARIABLES**
Time Allowed: **3 Hours**
Total: **70 Marks**
Instruction: **Answer Question One (1) and Any Other 4 Questions**

1. a. If $f(x,y) = \frac{xy^2}{x^2+y^2}$ does $\lim_{x\to 0, y\to 0} f(x,y)$ exist? **(5 marks)**
b. Find the derivative of $z = x^2 + 2xy + y^2$ **(5 marks)**
c. Define the following functions: (i) Constant function. (ii) Identity function (iii) Modulus function (iv) Square root function. (v) Trigonometric function. **(12 marks)**

2. a. If $f(x,y) = x^2 + x^3y^2 + y^4$. Find the partial derivatives of f with respect to x *and* y and compute the rates of change of the function in x *and* y directions at the point $-1,2)$ **(4 marks)**
b. Find the first order partial derivatives of $f(x,y) = x^4 + 6y^{0.5} - 10$ **(4 marks)**
c. Find $\lim_{(x,y)} \frac{x^2-y^2}{x^2+y^2}$ if it exist **(4 marks)**

3. a. Find the first order partial derivatives for
$$w = x^2y - 10y^2z^3 + 43x - 7\tan(4y)$$ **(4 marks)**
b. Define Curl. **(4 marks)**
c. Define a polynomial function of two variables. **(4 marks)**

4. a. Given $u = x^2 + 2y$ where $x = r\sin(t)$ and $\sin^2(t)$, determine the value of $\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial t}$ using the chain rule. **(6 marks)**
b. Define partial derivatives **(6 marks)**

5. a. Find the second partials and the cross partials of the function
$$z = 2x^3 + 3xy + 2y^2$$ **(6 marks)**

b. Using implicit differentiation Find

$\frac{d(x^3+y^3=6xy)}{dx}$ **(6 marks)**

6. a. Differentiate $x^2 - 2xy + y^3 = c$ **(4 marks)**

b. Prove that $\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$ **(4 marks)**