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NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 Examinations

Course Code:	**MTH 309**
Course Title:	**Optimization Theory**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**<u>Answer Question Number one</u> and any Other Four (4) Questions**

1. A bottling company operates two bottling plants. The growers are willing to supply fresh fruits in the following amounts:
   - S1: 200 tons at €11/ton
   - S2:310 tons at €10/ton
   - S3:420 tons at €9/ton

   Shipping cost in € per ton are:

To		Plant A	Plant B
From	S1	3	3.5
	S2	2	2.5
	S3	6	4

Plant capacities and labor costs are:

	Plant A	Plant B
Capacity	460 tons	560 ton
Labor cost	€26/tons	€21/ton

The bottled fruits are sold at €50/tons to the distributors. The company can sell at this price all they can produce. The objective is to find the best mixture of the quantities supplied by the three growers to the two plants so that the company maximizes its profits

(a). Formulate the problem as a linear program and explain it **(8 marks)**

(b). What assumptions have you made in expressing the problem as a LP (**5 marks**)

(c). Explain the meaning of the dual values associated with the supply plant capacities **(3 marks)**

(d). Given the Objective function $Maximize\ z = x_1 + 4x_2 + 7x_3$. Generate the simplex tableau associated with the basis $B = \begin{pmatrix} 1 & 3 \\ 2 & -2 \end{pmatrix}, b = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ **(6 marks)**

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2. (a). Solve by graphical methods the following LP problem,

$$\begin{aligned} &Minimize \quad Z = 3x_1 + 2x_2 \\ &\text{Subject to} \quad 3x_1 + x_2 \geq 3 \\ &\qquad 4x_1 + 3x_2 \geq 6 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

(4 marks)

(b). Find the dual and hence the primal solution of the LP problem in (a) above.

(8 marks)

3. Three electric power plants with capacities of 25, 40, and 30 million Kwh, supply electricity to three towns. The maximum demands at the three towns are estimated at 30, 35, and 30 million Kwh. The price per million Kwh at the three towns is given in the table below.

	Towns		
	1	**2**	**3**
1	\$600	\$700	\$400
2	\$320	\$300	\$350
3	\$500	\$400	\$450

(a). Formulate the problem as a transportation problem **(4 marks)**

(b). Solve the problem to optimality starting with a North-West corner method.

(8 marks)

4. (a). Determine the convexity, concavity, or saddle points of the function given by

$$f(x_1, x_2, x_3) = 3x_1^2 + 2x_2^2 + 3x_3^2 - 5x_1x_2 - 6x_2x_3 + 5x_2x_3 - 6x_1 - 4x_2 - 9x_3$$

(4 marks)

(b). Optimize $f(x_1, x_2, x_3) = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$

$$\begin{aligned} &\text{Subject to} \quad x_1 + x_2 + x_3 = 15 \\ &\qquad 2x_1 - x_2 + 2x_3 = 20 \\ &\qquad x_1, x_2, x_3 \geq 0 \end{aligned}$$

(8 marks)

5. Solve the following problem by Gomory cutting plane method

$$\begin{aligned} &\text{Maximize} \quad z = 5x_1 + 6x_2 \\ &\text{Subject to} \quad x_1 + 2x_2 \leq 7 \\ &\qquad 2x_1 - x_2 \leq 3 \\ &\qquad x_1, x_2 \in \mathbb{Z}_+ \end{aligned}$$

(12 marks)

6. Use the Kuhn -Tucker conditions to Optimize:

$$Z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$$

$$\begin{aligned} &\text{subject to} \quad x_1 + x_2 \leq 2 \\ &\qquad 2x_1 + 3x_2 \leq 12, \\ &\qquad x_1, x_2, x_3 \geq 0 \end{aligned}$$

(12 marks)