



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 Examinations

Course Code: MTH304
Course Title: COMPLEX ANALYSIS
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 4 Questions

1a. Suppose the function f given by $f(z) = u(x, y) + iv(x, y)$ has a derivative at $z = z_0 = (x_0, y_0)$. Derive Cauchy Riemann Equations. **(8 marks)**

1b. find all points at $z = x + iy$ which the function f given by $f(z) = x^3 - i(1 - y)^3$ is differentiable. **(8 marks)**

1c. If $z = x + jy$, find the locus defined as $|z| = 5$. **(6 marks)**

2a. If $z = \frac{1}{2-j3} + \frac{1}{1-j2}$ express z in terms of $a + jb$ **(4 marks)**

2b. Write in polar form $re^{i\theta}$ (i) $3 + 4j$ (ii) $12 + 5j$ **(4 marks)**

2c. If $z = x + iy$, find the equation of the locus $\left| \frac{z+1}{z-1} \right| = 2$ **(4 marks)**

3a. What is a vector valued function? **(4 marks)**

3b. Find a polar form of $(1 + i)(1 + i\sqrt{3})$ **(4 marks)**

3c. Suppose $f(z) = 2z^2$. What is $\lim_{z \rightarrow z_0} \frac{f(z) - f(z_0)}{z - z_0}$ **(4 marks)**

4a. (i) What are conjugates? (2 marks)

(ii) $(u, v) + (a, v) = (x, y)$ (iii) $(8,1) + (x, y) = (10,1)$ (2 marks)

4b. i. What are the products of the following? (2 marks)

(ii) $(4 + 3j)(4 - 3j)$ (iii) $(x - jy)(x + jy)$ (2 marks)

4c. When are two complex numbers said to be equal? (4 marks)

5a. Let c be the circle $|z|=4$. Evaluate the integral $\int_c \frac{\cos z}{z^2-6z+5} dz$ (4 marks)

5b. Define Cauchy integral formula (4 marks)

5c. if $z = x + jy$, find the equation of the locus $\arg(z^2) = -\frac{\pi}{4}$ (4 marks)

6a. Find the $\lim_{x \rightarrow 0} \left\{ \frac{\tan x - x}{x^3} \right\}$ (4 marks)

6b. What Is an entire function? (4 marks)

6c. Compute $\int_C \frac{1}{z^2+4} dz$ over the contour C shown below (4 marks)

