



NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 Examinations..

Course Code: MTH302
Course Title: Elementary Differential Equations II
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 4 Questions

1. (a) Find a Fourier cosine series for $f(x) = e^x$ on $(0, \pi)$. **(12 marks)**
 (b) Find a recurrence formula for the power series solution around $x = 0$ for the nonhomogeneous differential equation $(x^2 + 4)y'' + xy' = x + 2$. **(10 marks)**
2. Determine whether $x = 0$ is a regular singular point of the following differential equations
 - (i) $2x^2y'' + 7x(x + 1)y' - 3y = 0$ **(6 marks)**
 - (ii) $x^3y'' + 2x^2y' + y = 0$ **(6 marks)**
3. (a) Determine whether $x = 0$ is a regular singular point of the following differential equations

$$8x^2y'' + 10xy' + (x - 1)y = 0$$
 (5 marks)
 (b) Verify the orthogonality property for the Sturm-Liouville problem $y'' + \lambda y = 0; y(0) = 0, y(1) = 0$ if the problem has the eigenvalues $\lambda_n = n^2\pi^2$ corresponding to the eigenfunctions $y_n(x) = A_n \sin n\pi x$, $n = 1, 2, \dots$ **(7 marks)**
4. Find a Fourier sine series for $f(x) = \begin{cases} 0 & x \leq 2 \\ 2 & x > 2 \end{cases}$ on $(0, 3)$. **(12 marks)**
5. Find a Fourier sine series for $f(x) = e^x$ on $(0, \pi)$. **(12 marks)**
6. Find the first four terms in each portion of the series solution around $x_0 = 0$ for the following differential equation $(x^2 + 1)y'' - 4xy' + 6y = 0$ **(12 marks)**