NATIONAL OPEN UNIVERSITY OF NIGERIA
PLOT 91, CADASTRAL ZONE, NNAMDI AZIKIWE EXPRESSWAY, JABI - ABUJA
FACULTY OF SCIENCES
DEPARTMENT OF PURE AND APPLIED SCIENCES
SEPTEMBER, 2020_1 EXAMINATION

COURSE CODE: **PHY313**
COURSE TITLE: **MATHEMATICAL METHOD FOR PHYSICS II**
CREDIT UNIT **3**
TIME ALLOWED **(2½ HRS)**

INSTRUCTION: ***Answer question 1 and any other four questions***

1. a. Obtain the $\frac{df}{dz}$ of the function $f(z) = 4x + y + i(-x + 4y)$ [3 marks]

b. Evaluate the continuity of the followings: (i) $f(z) = \begin{cases} \frac{z^3 - iz \pm i}{z - i}, & z \neq i \\ 0 & z = 0 \end{cases}$ at $z = i$ [3 marks]

c. Obtain the limit of the following function $\lim_{z \to 1+i} \frac{z^2 - z + 1 - i}{z^2 - 2z + 2}$ [3 marks]

d. Evaluate the following integral using Residue theorem $\int_c \frac{1+z}{z(2-z)} dz$ [3 marks]

e. Obtain the residue of $\frac{1}{(z^2+1)^3}$ at z $= i$ [3 marks]

f. State three rules in obtaining Singularity. [3 marks]

g. Obtain the singularities of the following functions: $(i)\ f(z) = \sin \frac{1}{z}$ $(ii)\ g(z) = \frac{e^{1/z}}{z^2}$
[4 marks]

2. a. Show that $f(z) = e^z$ is analytic and that $\frac{de^z}{dz} = e^z$ [4 marks]

b. Using the complex Integral, show that $\int_0^1 (t-1)^3 dt = -\frac{5}{4}$ [4 marks]

c. Show that $\int_0^{\frac{\pi}{2}} e^{t+it} dt = \frac{1}{2}\left(e^{\pi/2} - 1\right) + \frac{i}{2}\left(e^{\pi/2} + 1\right)$ [4 marks]

3. a. Determine the pole of the function $f(z) = \frac{1}{z^4 + 1}$ [4 marks]

b. Define is an Analytic function in a domain, hence write an expression for a function *f(z)* to be analytic.
[4 marks]

c. from equation 1e above, show that *f(z)* satisfies Laplace's equation. [4 marks]

4. a. State Cauchy Residue Theorem [4 marks]

b. Show that the function $f(z)=|z|^2$ is continuous everywhere but not differentiable except at the origin. [4 marks]

c. Using Residue theorem, evaluate $\frac{1}{2\pi i}\int_c \frac{e^{zt}dz}{z^2(z^2+2z+2)}$ where C is the circle $|z|=3$. [4 marks]

5. a. If a function *f(z)* is analytic in a region R, then obtain its derivative at any point z = a of R [4 marks]

b. Prove that $\int_C \frac{dz}{z-a}=2\pi i$, where C is the circle $|z-a|=$ r [4 marks]

c . Use Cauchy's integral formula to evaluate $\int_C \frac{z}{(z^2-3z+2)}dz$ where c is the circle $|z-2|=\frac{1}{2}$

[4 marks]

6. a. Obtain the residue of $f(z)=\frac{ze^z}{(z-a)^3}$ at its pole [4 marks]

b. Use the complex variable technique to obtain the value of the integral $\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}$ [4 marks]

c . Evaluate $\int_0^{\infty}\frac{\cos mx}{(x^2+1)}\,dx$ [4 marks]