NATIONAL OPEN UNIVERSITY OF NIGERIA
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2020_1 EXAMINATION

Course Code: **MTH 423**
Course Title: **Integral Equations**
Credit Unit: **3**
Time Allowed: **3 Hours**
Instruction: **Answer Number One (1) and Any Other Four (4) Questions**

Q1. (a) Solve the integral equation $A(x) = x + 1 + \int_0^x (1 + 2(x - y))d(y)dy.$ **(8 marks)**

(b) Transform $\frac{d^2y}{dx^2} + \lambda y = 0$ into an integral equation with $y = 0$ at $x = 0$ and $y' = 0$ at $x = 1$ **(8 marks)**

(c) State Hilbert-Schmidt theorem **(6 marks)**

Q2. (a) Solve the equation

$$y'' + 2xy' + y = 0,\ y(0) = 1,\ y'(0) = 0.$$ **(4 marks)**

(b) If $Q(x) = 3\int_0^x \cos(x - y)\, Q(y)dy + e^x$. Find the possible solution to the equation **(8 marks)**

Q3. (a) State the system of Laplace transform **(4 marks)**

(b) Define positive Kernel. **(3 marks)**

(c) Show by using Laplace transform the statement of kernel of the convolution type. **(5 marks)**

Q4. (a) If $K(x, y)$ is symmetric and continuous. Show that K has at least one Eigenvalue. **(10 marks)**

(b) Define half-plane of convergence. **(2 marks)**

Q5. (a) Solve the equation $\phi(x) = \lambda \int_0^1 (1 + xt)\, \phi(t)dt, \qquad 0 \leq x \leq 1.$ **(6 marks)**

(b) Write the general method of solution for Fredholm equation with degenerate kernel

(6 marks)

Q6. (a) Use Laplace transform to solve the integral equation

$$f(x) = \int_0^x \mathrm{K}(x - y)\, f(y)dy + g(x)$$ **(6 marks)**

(b) If $K(x, y)$ is symmetric and continuous. Write the three (3) necessary conditions for K

(3 marks)

(c) State Convolution system of integral equation **(3 marks)**