NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
September Examination, 2020_1

Course Code: MTH 411
Course Title: Measure Theory and Integration
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Attempt Number One (1) and Any Other Four (4) Questions

1. (a) Describe the greatest lower bound of the measures of all bounded open sets containing set G. **(1 mark)**
   (b) Let G_1 , G_2 be open sets such that $G_1 \subseteq G_2$, prove that $m(G_1) \leq m(G_2)$. **(5 marks)**
   (c) Let X be an arbitrary set. State the properties of the collection **Ω** of subsets of X to be called an algebra. **(4 marks)**
   (d) Write on measurable function f: A → [-∞, +∞] by stating the four conditions f must satisfy. **(4 marks)**
   (e) Distinguish between a measurable function and a Borel function using four examples. **(8 marks)**

2. Let (X, $\mathscr{M}$) be a measurable space, and let fl be a finitely additive measure on (X, $\mathscr{M}$). Show that fl is a measure if either
   (i) $\lim_k$ fl(A_k) = fl($\bigcup_k A_k$) holds for each increasing sequence {A_k } of sets that belong to $\mathscr{M}$. Or
   (ii) $\lim_k$ fl(A_k) = 0 holds for each decreasing sequence {A_k} of sets that belong to $\mathscr{M}$ and satisfy $\bigcap_k A_k = \emptyset$. **(12 marks)**

3. (a) When is S: X → $\mathbb{R}$ a simple function? **(2 marks)**
   (b) State Beppo Levi's theorem. **(4 marks)**
   (c) Let (X, $\mathscr{M}$) be a measurable space, let A be a subset of X that belongs to $\mathscr{M}$, and let f and g be [-∞, +∞] - valued measurable functions on A.
   Show that $f \vee g$ and $f \wedge g$ are measurable. **(6 marks)**

4. (a) State (i) Monotone Convergence theorem. **(3 marks)**
   (ii) Dominated Convergence theorem. **(4 marks)**
   (b) Let (X, $\mathscr{M}$, fl) be a measure space, and let f and g be extended real-valued functions on X that are equal almost everywhere. If fl is complete and if f is measurable, explain What is meant by g is measurable. **(5 marks)**

5. Let (X, $\mathscr{M}$, fl) be a measure space. Explain the properties that hold almost everywhere. **(12 marks)**

6. (a) Define equivalent functions. **(3 marks)**
   (b) Let { n } be a sequence of measurable functions. n : X → C a.e. Suppose that $\Sigma_{n=1} \int_x$ | n | d fl < ∞. Show that $\Sigma_{n=1}$ $_n$(x) converges to f(x) a.e on X. **(9 marks)**