



NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
September, Examination 2020_1

Course Code: MTH401

Course Title: General Topology I

Credit Unit: 3

Time Allowed: 3 Hours

Instruction: Attempt Number One (1) and Any Other Four (4) Questions

1. (a) Show that $d(x,y) = |x-y|^3$ does not define a metric on \mathbb{R} . **(4 marks)**
(b) Let \mathbb{R}^2 be the set of all ordered pairs of real numbers endowed with the metric $d_1(x, y) = \sum_{i=1}^2 |x_i - y_i|$ for arbitrary $x = (x_1, x_2)$ and $y = (y_1, y_2)$ in \mathbb{R}^2 . Describe the open ball $B((0, 0); 1)$. **(3 marks)**
(c) Why is it that any open ball centered at $0 \in [0, 1)$ with the usual metric on \mathbb{R} , not open in \mathbb{R} ? **(2 marks)**
(d) Let $E = \mathbb{R}$ and $A = [0, 1)$. Now, endow \mathbb{R} with the following metric: for all $x, y \in \mathbb{R}$ $d(x, y) = 1$, if $x \neq y$ and $d(x, y) = 0$ if $x = y$ with the usual metric. Show that $A = [0, 1)$ is open in \mathbb{R} . **(9 marks)**
(e) Explain function space with the metric defined on it. **(4 marks)**
2. Show that in any metric space (E, d) , each open ball is an open set in E . **(12 marks)**
3. Show that the interval $[a, b]$ is closed in \mathbb{R} . Illustrate with an example. **(12 marks)**
4. Let $E = \mathbb{R}$ (the reals) and let d_0 be defined by $d_0(x, y) = 1$, if $x \neq y$; $d_0(x, y) = 0$ if $x = y$, and let $F = [0, 1)$. Show that:
(i) F has no limit points. **(6 marks)**
(ii) F is closed. **(4 marks)**
(iii) F is open. **(2 marks)**
5. Let $\{F_i\}_{i \in I}$ be a nonempty family of closed sets of a metric space (E, d) . Show that (i) $\bigcap_{i \in I} F_i$ is closed in E . **(6 marks)**
(ii) $\bigcup_{i=1}^k F_i$ is closed in E . **(6 marks)**
6. (a) Show that a subset F of a metric space (E, d) is closed in E if and only if its complement is open in E . **(6 marks)**
(b) Show that every singleton subset of any metric space is closed. Hence, every finite set is closed. **(6 marks)**