NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
September Examination 2020_1

Course Code: MTH 381
Course Title: Mathematical Methods III
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Answer Question Number One and Any other Four Questions.

1. a) If $x = r\cos\theta$ and $y = r\sin\theta$, evaluate $\dfrac{\partial(x, y)}{\partial(r, \theta)}$. **[5 Marks]**

b) Evaluate $\int_0^1 dx \int_0^x e^{\frac{y}{x}} dy$ **[4 Marks]**

c) Calculate the *curl* of the vector $\vec{f} = xyzi + 3x^2y\,j + (xz^2 - y^2z)k$ **[4 Marks]**

d) Show that the function $e^x(\cos y + i\sin y)$ is an analytic function, find its derivative. **[5 Marks]**

e) Find the *Laplace* transform of $\dfrac{\sin 2t}{t}$. **[4 Marks]**

2. a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y} + \dfrac{\partial}{\partial z}\right)^2 = -\dfrac{9}{(x+y+z)^2}$ **[7 Marks]**

b) Using Stoke's theorem or otherwise, evaluate $\int_C \left[(2x - y)dx - yz^2\,dy - y^2z\,dz\right]$ where C is the circle $x^2 + y^2 = 1$, corresponding to the surface of sphere of unit radius. **[5 Marks]**

3. a) Compute $\iiint \dfrac{dxdydz}{(x+y+z+1)^3}$, if the region of integration is bounded by the coordinate planes and the plane is $x + y + z = 1$. **[7 Marks]**

b) Find the complex Fourier transform of Dirac delta function $\delta(t - a)$. **[5 Marks]**

4. a) Find the Fourier series representing $f(x) = x, \quad 0 < x < 2\pi$ **[8 Marks]**

b) Find the inverse Laplace transform of $\dfrac{1}{s^2 + 25}$. **[4 Marks]**

5. a) Evaluate $\int_{1+i}^{2+4i} z^2 dz$

(i) along the parabola $x = t$, $y = t^2$ where $1 \le t \le 2$. **[4 Marks]**

(ii) along the straight line joining the line $1 + i$ and $2 + 4i$ **[4 Marks]**

b) Find the finite Fourier sine and cosine transform of $f(x) = 1$ in $(0, \pi)$. **[4 Marks]**

6. a) Evaluate $\iint_{\Re} \sqrt{x^2 + y^2}\, dxdy$, where $\Re$ is the region bounded by $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$ **[6 Marks]**

b) Determine the residues of $\frac{z^2}{(z-2)(z^2+1)}$ at each simple pole. **[6 Marks]**