NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
June Examination 2020

Course Code: MTH341
Course Title: Real Analysis
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Answer Number One (1) and Any Other Four (4) Questions

1. Define the following:
   i. Derivative at a point **(2 marks)**
   ii. Derivative in an interval **(2 marks)**

b. Let $f : R \to R$ be a function defined as $f(x) = x^2 \cos(1/x)$ if $x \neq 0$ and $f(0) = 0$. Find the derivative of f at x = 0, if it exists. **(6 marks)**

c. Let a function $f : [0,5] \to R$ be defined as $f(x) = \begin{pmatrix} 2x+1, 0 \le x \le 3 \\ x^2 - 2, 3 \le x \le 5 \end{pmatrix}$ is

f derivable at x = 3 **(6 marks)**

d. Show that the function f defined on R by $f(x) = x^3 - 3x^2 + 3x - 5$ for all

$x \in R$ is increasing in every interval. **(6 marks)**

2. Separate the intervals in which the function f defined on R by $f(x) = 2x^3 - 15x^2 + 36x + 5$ for all $x \in R$ is increasing in every interval **(8 marks)**

b. Let $f : R \to R$ be a continuous function defined on R. Show that f

is differentiable on R **(4 marks)**

3. Verify the Rolle's theorem for the function defined by:

   i. $f(x) = x^3 - 6x^2 + 11x - 6$ for all $x \in [1,3]$ **(6 marks)**

   ii. $f(x) = (x-a)^m (x-b)^n$ for all $x \in [a,b]$ where m and n are positive integers **(6 marks)**

4. Show that there is no real number λ, for which the equation $f(x) = x^3 - 27x + \lambda = 0$ has two distinct roots in [0,2] **(12 marks)**

5. Let f be the function defined on [-1,2] as $f(x) = |x|$. Find the derivative of f **(6 Marks)**

b. Verify the hypothesis and conclusion of Langrange's mean value theorem for the functions defined as $f(x) = \frac{1}{x}$ for all $x \in [1,4]$ **(6 marks)**

6. Apply Cauchy's mean value theorem to the functions f and g defined as $f(x) = x^2, g(x) = x$ for all $x \in [a,b]$. **(6 marks)**

b. Show that $\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta$. where $0 < \alpha < \theta < \beta < \pi/2$ **(6 marks)**