NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.
FACULTY OF SCIENCES
DEPARTMENT OFMATHEMATICS
September Examination, 2020_1

Course Code: **MTH 315**
Course Title: **Analytic Dynamics**
Credit Unit: **3**
Time Allowed: **3 Hour**
Instruction: **Answer Question Number one and any Other Four Questions**

1. (a) Define the following:
   (i) Total kinetic energy of a system of N particles **(2 marks)**
   (ii) Centre of mass of a system of N particles **(2 marks)**
   (iii) When does the total angular momentum is said to be conserved? **(3 marks)**
   (b) (i) Define the rest frame of a rigid body. **(2 marks)**
   (ii) Prove that if P and Q are two points fixed a rigid body and r is the vector from P to Q, then the velocities V_P and V_q of P and Qrelative to a frame $\tilde{R}$ are related by $V_P = V_Q + w_r$ when ω_n the angular velocity of the body relative to $\tilde{R}$ **(4 marks)**

   (c) (i) Discuss the motion of a body that glides frictionless on a uniformly rotating wire, r is its distance from the center of rotation. Given are the initial conditions: $r(t=0) = r_0, \dot{r}(t=0) = -r_0\boldsymbol{\omega}$,$\omega = constant\ angular\ velocity\ of\ the\ wire$ **(3 marks)**
   (ii) A parabolically curved wire rotates with constant angular velocity **ω** around the $z\ axis$. on this rotating wire, a bead of mass m moves frictionless in the earth's gravitational field ($g = -ge_z$). If the wire is just within the yz plane then it holds for the position of the mass $z = \alpha y^2$ $(\alpha > 0)$
      (a) Find the constraints **(2 marks)**
      (b) How many degrees of freedom are left ? **(2 marks)**
      (c) Use cylindrical coordinates (ρ, φ, z) to represent the lagrangian. **(2 marks)**

2. Consider a planar thread pendulum with the thread length l in the homogeneous gravitational field only small deflections of the pendulum are to be discussed.
   (a) Find the langrangian and the equationof motion, choose the initial conditions such that at time $t = 0$ the pendulum swings through its equilibrium position, how big is the frequency ω_0 of the oscillation ? **(4 marks)**
   (b) Calculate the thread tension **(4 marks)**
   (c) Show that $F = x^2 y z_i - xy{z_k}^2$ is non-conservative **(4 marks)**

3. (a) Define the following terms:
   (i) Amplitude of a motion **(2 marks)**
   (ii) Period of a motion **(2 marks)**
   (iii) Simple Harmonic motion **(2 marks)**

   (b) A particle is moving with simple harmonic motion of period 4π about a centre 0, it passes through a point distance $4m$ from 0. Find the time which elapses before it next passes through this point. **(6 marks)**

4. (a) One end of an elastic string of length $24cm$ is fixed ended and to the other suspended end, a mass of $5kg$ is attached , which when in equilibrium stretches the string $4cm$. The mass is pulled down at a distance of $3cm$ below its equilibrium position and then released. Find the period of oscillation and the maximum kinetic energy of the mass. **(6 marks)**

   (b) Three forces of magnitude $15Q, 10Q, 5Q$ act on a particle in directions which make 120^0 with one another. Find their resultant. **(6 marks)**

5. (a) Due to a force field, a particle of mass 5 units moves alone a space curve whose position vector is given as a function of time t by $\mathfrak{r} = (2t^2 + t)i + (3t^4 - t^2 + 8)j - 12t^2k$
   Find:
   (i) the velocity, **(2 marks)**
   (ii) the momentum **(2 marks)**
   (iii) the acceleration and **(2 marks)**
   (iv) the force field at any time t. **(2 marks)**

   (b) A particle of mass m moves in the xy plane so that its position vector is
   $r = acoswti + bsinwtj$
   Where a, b and w are positive constant and $a > b$.
   (i). Show that the particle moves in an ellipse. **(2 marks)**
   (ii). Show that the force acting on the particle is always directed toward the origin. **(2 marks)**

6. Show that for the functions
$$f = f(\boldsymbol{q}, \boldsymbol{p}, t); \quad g = g(\boldsymbol{q}, \boldsymbol{p}, t) \quad h = (\boldsymbol{q}, \boldsymbol{p}, t)$$
   the following relations are valid:
   i. $\frac{\partial}{\partial t}\{f, g\} = \left\{\frac{\partial f}{\partial t}.g\right\} + \left\{f.\frac{\partial g}{\partial t}\right\}$ **(4 marks)**

   ii. $\frac{d}{dt}\{f.g\} = \left\{\frac{df}{dt}.g\right\} + \left\{f.\frac{dg}{dt}\right\}$ **(4 marks)**

   iii. $\{f, g.h\} = g\{f, h\} + \{f, g\}h$ **(4 marks)**