



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

**FACULTY OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**

**Course Code:** MTH315  
**Course Title:** Analytical Dynamics  
**Credit Unit:** 3  
**Time Allowed:** 3 Hours  
**Total:** 70 Marks  
**Instruction:** Answer Question One and Any Other Four Questions

1. (a) Determine the number of degrees of freedom in each of the following cases:
  - (i) Seven particles moving freely in a plane **(2 marks)**
  - (ii) Ten particles moving freely in space. **(2 marks)**
- (b) A system of particles consists of a 3 gram mass located at (2, 0, -1), a 5 gram mass at (-5,1,3) and 2 gram mass at (3, -1, 1). Find the center of mass. **(6 marks)**
- (c) A uniform beam is 72m long and has a mass 200kg and masses of 120kg and 160 kg are suspended from its ends; at what point must the beam be supported so that it may rest horizontally? **(8 marks)**
- (d) A pentagon QRSXW has masses 7, 1, 5, 2 and 3 units located at its vertices Q(1,2,-1), R(3,-2,2), S(2,-2,3), X(1,-2,4) and W(4,1,3). Find the coordinates of the center of mass. **(4 marks)**
2. Three particles of masses 1,3,6 respectively have position vectors
 
$$r_1 = (2t + 2)i - 11t^2j + (t^3 + 6t - 10)k,$$

$$r_2 = -5ti + 3t^2j + k,$$

$$r_3 = \frac{1}{3}t^3i - tj + 2tk, \text{ where } t \text{ is time. Find}$$
  - (a) The velocity of the center of mass at time  $t = 1$  **(8 marks)**
  - (b) The acceleration at  $t=1$ . **(4 marks)**
3. A particle of mass 5 units moves along a space curve whose position vector is given as a function of time  $t$  by
 
$$r = (6t^3 - t + 2)i + (2t^2 - t)j + (3t^3)k$$

At time  $t = 2$ , find the

  - (a) momentum **(7 marks)**

(b) force field. (5 marks)

4. A particle of mass 2 moves in a force field depending on time  $t$  given by

$F = 42t^2i - 8tj + 4tk$ . Assuming that at  $t = 0$  the particle is located at

$r_0 = i - 2j + k$  and has velocity  $v_0 = 2i + 3j - 4k$ , find

(a) the velocity, (6 marks)

(b) the position at any time  $t$ . (6 marks)

5. A particle moves along the  $x$  axis in a force field having potential

$$V = \frac{\alpha}{3}x^3 - \frac{\beta}{3}x^2 + 2\gamma x + 10\delta,$$

where  $\alpha, \beta, \gamma$  and  $\delta$  are positive constants.

Determine the point(s) of equilibrium. (12marks)

6. (a) State without proof the Liouville's theorem. (4marks)

(b) Minimise the integral

$$I = \int_0^{\frac{\pi}{2}} \left[ \left( \frac{dy}{dt} \right)^2 - y^2 + 2ty \right] dt, y(0) = 0 \text{ and } y\left(\frac{\pi}{2}\right) = 0 \quad \text{(8 marks)}$$