

## NATIONAL OPEN UNIVERSITY OF NIGERIA

## Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja FACULTY OF SCIENCES <br> DEPARTMENT OF MATHEMATICS <br> SEPTEMBER 2020_1 EXAMINATION

## Course Code: MTH 311

## Course Title: Calculus of Several Variables

## Credit Unit: 3

Time Allowed: 3 Hours
Instruction: Answer Question Number One and Any other Four Questions
1 a) Investigate $\lim _{h \rightarrow 0} \frac{\sin (3 h)}{h}$
b) Find the extrema value of $f(x, y)=x^{2}-8 \ln x$ at $[1,4]$
[3 Marks]
[5 Marks]
c) Check that $\frac{\partial^{2} f}{\partial u \partial t}=\frac{\partial^{2} f}{\partial t \partial u}$ for $f=e^{\frac{u}{t}}$
[4 Marks]
d) Write out the Langrage equation for $f(x, y)=2 x+5 y$ on the ellipse $\left(\frac{x}{4}\right)^{2}+\left(\frac{y}{3}\right)^{2}=1$
[6 Marks]
e) Write $h(x, y)=e^{-x^{3} y}$ as a composite function and evaluate $\lim _{(x, y) \rightarrow(1,2)} h(x, y)$
2. a) Calculate the second - order partial of $f(x, y)=x^{3}+y^{2} e^{x}$
[5 Marks]
b) The altitude of a mountain at $(x, y)$ is $f(x, y)=2500+100\left(x+y^{2}\right) e^{-0.3 y^{2}}$. Find the directional derivative of $f$ at $P=(-1,-1)$ in the direction of unit vector $u$ making an angle of $\frac{\theta}{4}$ with the gradient.
[7 Marks]
3a) Find the critical points of $f(x, y)=\left(x^{2}+y^{2}\right) e^{-x}$ and analyze them using the second derivative test.
[8 Marks]
b) Let $f(x, y)$ be a function of two variables, and let $(r, \theta)$ be polar coordinate, express $\frac{\partial f}{\partial \theta}$ in terms

$$
\text { of } \frac{\partial f}{\partial x} \text { and } \frac{\partial f}{\partial y}
$$

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4. a) Find the maximum and minimum value of $f(x, y)=81 x^{2}+y^{2}$ subject to the constraint

$$
4 x^{2}+y^{2}=9
$$

b) Find the dimension of the box with largest volume if the total surface area is $32 \mathrm{~cm}^{2}$.
5. a) If $x=u-v+w, y=u^{2}-v^{2}-w^{2}$ and $z=u^{3}+v$, Find Jacobian $\frac{\partial(x, y, z)}{\partial(u, v, w)}$
b) If $f(x, y)=5 x-3 y$ subject to the constraint $x^{2}+y^{2}=136$
(i) Write out the Langrage equation
(ii) Find the maximum and minimum value of $f(x, y)$
6. a) If $f(x, y)=x^{3} y^{2}$, find $\frac{d f}{d t}$ if $x^{5}+y=t$ and $x^{2}+y^{3}=t^{3}$
[4 Marks]
b) Find the maximum and minimum of $f(x, y, z)=4 y-2 z$ subject to the constraints $2 x-y-z=2$ and $x^{2}+y^{2}=1$

