NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
September 2020_1 Examination

Course Code: MTH 309
Course Title: Optimization Theory
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Answer Question Number One and Any Other Four Questions

1 (a) Define clearly the following terms:

(i) Feasible Solution.
(ii) Basic feasible solution.
(iii) Non-degenerate feasible solution.
(iv) Degenerate basic feasible solution.
(v) Convex set.
(vi) Convex function. **(12 marks)**

(b) Consider the following problem.

$$\text{Maximize } 2x_1 + 5x_2$$

$$\text{Subject to: } x_1 + 2x_2 \leq 16$$

$$2x_1 + x_2 \leq 12$$

$$\text{With } x_1, x_2 \geq 0$$

(i) Sketch the feasible region in the (x_1, x_2) space.
(ii) Identify the region in the (x_1, x_2) space where the slack variable x_3 *and* x_4 are equal to zero.
(iii) Solve the problem using graphical method. **(10 marks**

2 (a) State five important results in duality. **(5 marks)**

(b) Solve the following LPP

$$\text{Maximize } z = 6x_1 + 8x_2$$

$$\text{Subject to: } 5x_1 + 2x_2 \leq 20$$

$$x_1 + 2x_2 \leq 10$$

$$\text{With } \; x_1, x_2 \geq 0$$

by solving its dual problem. **(7 marks)**

3 Define and give an example (with justification) of the following in $\mathbb{R}^n$.

(a) Continuous function

(b) Differentiablity

(c) Partial derivative

(d) Directional derivative. **(12 marks)**

4 (a) When is a quadratic form said to be:

(i) Positive semidefinite; **(2 marks)**

(ii) Negative semidefinite. **(3 marks)**

(b) Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ be a function which satisfies $f(x+y) = f(x)f(y) \; \forall \, x, y \; \in \mathbb{R}$

(i) Show that if f is continuous at $x = 0$, then it is continuous at every point of $\mathbb{R}$

(ii) Show that $\forall \, m \in \mathbb{Z}$; $f(mx) = (f(x))^m$. **(7 marks)**

5 (a) State (without proof) the Weierstrass theorem. **(4 marks)**

(b) Let $f: D \longrightarrow \mathbb{R}$, where D is a nonempty closed subset of $\mathbb{R}^n$. Prove that if f is coercive and continuous on some open set containing D then

(i) the function f is bounded below on D.

(ii) any minimizing sequence of f in D is bounded. **(8 marks)**

6 (a) Find the minimum and maximum of $f(x, y) = x^2 - y^2$ on the unit circle $x^2 + y^2 = 1$ using the Lagrange multipliers method. **(5 marks)**

(b) Find the Local and global minimizers and maximizers of the $f(x) = x^2 e^{-x^2}$.

(4 marks)

(c) State any theorem that links convexity and optimization. **(3 marks)**