



## NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

### FACULTY OF SCIENCES DEPARTMENT OF MATHEMATICS

**Course Code:** MTH 308

**Course Title:** Introduction to Mathematical Modelling

**Credit Unit:** 3

**Time Allowed:** 3 Hours

**Total:** 70 Marks

**Instruction:** Answer Question Number One and Any Other Four Questions

1. (a) By the use of mathematical expressions, write down the dimensional formulas of the following:

- i. Velocity (2 marks)
- ii. Acceleration (2 marks)
- iii. Force (2 marks)
- iv. Work-done (2 marks)

- (b) i. Given that  $\frac{dQ}{dt} = kQ$

with solution,  $Q = Q_0 e^{-kt}$  where  $Q_0$  is the amount present at time  $t = 0$ .  $k$  is a proportionality constant. What is the dimension of  $k$ ? (2 marks)

- ii. Interpret the solution obtained for a formulation of the model of a simple pendulum given as:

$$T_0 = 2\pi \sqrt{\frac{l}{g}} \quad (3 \text{ marks})$$

- (c) Explain the following steps involved in problem formulation:

- i. Stating the question (3 marks)
- ii. Identifying the relevant factors (3 marks)
- iii. Mathematical description (3 marks)

2. Explain the following concepts as they relate to Mathematical Modeling with relevant examples

- (a) Empirical Models (6 marks)
- (b) Theoretical Model (6 marks)

3. (a) A formulation of the simple pendulum model involves the two differential equations

$$m \frac{d^2x}{dt^2} = -T \sin \theta$$

$$m \frac{d^2y}{dt^2} = T \cos\theta - mg$$

Where  $T$  and  $\theta$  are unknown. Obtain a non-linear differential equation in terms of  $T$  and  $\theta$   
**(8 marks)**

(b) With the usual notation of the simple pendulum model, find  $T_0$  if  $\theta_0 = 20^\circ$   
given that  $l = 20 \text{ cm}$  and  $g = 980 \text{ cm/sec}^2$  **(4 marks)**

4. (a) Briefly explain the type of modelling will you use for the launching of a rocket/satellite for meteorological purposes. **(4marks)**  
(b) Briefly explain the following processes in Mathematical Modelling.  
(i) Validation of a model. **(4 marks)**  
(ii) Interpreting the solution of a model **(4 marks)**
5. (a) (i) A raindrop, beginning at rest, falls from a cloud 705.6 m above the ground. By adopting a well-known model, determine how long it takes for the raindrop to reach the ground?  
**(4 marks)**  
(ii) State two things that could contradict the adopted model in (i) above **(4 marks)**  
(b) Explain the concept of evaluation of a mathematical model. **(4 marks)**
6. (a) Suppose that  $D_t$ ,  $S_t$ , and  $p_t$  represent the respective demand, supply and price at time  $t$  respectively, formulate a difference equation for the dynamic stability of market equilibrium with  $D_t$  and  $S_t$  given as:  
$$D_t = ap_t + b$$
$$S_t = Ap_t + B$$
Where  $a, b, A$  and  $B$  are all constants **(8 marks)**  
(b) List four main classification of Mathematical Models **(4 marks)**