NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS

Course Code: MTH 305
Course Title: Complex Analysis II
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Answer Question Number One and Any other Four Questions.

1a) Express $z = -\sqrt{6} - \sqrt{2}\,i$ in polar form. [4 Marks]

b) If $f(z) = z^2$, prove that $\lim_{z \to z_0} z_0^2$. [4 Marks]

c) Find the analytic function $f(z) = u + iv$, given that $u = e^{-x}(x \sin y - y \cos y)$. [4 Marks]

d) If $A(x, y) = 2xy - i\,x^2y^3$, find $div\ A$. [4 Marks]

e) Find the residue of $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane. [6 Marks]

2. a) Given the complex function $f(z) = \dfrac{1}{(z^2+4)}$. Find the first four terms of the Taylor series expansion $f(z)$ about $z = -i$. [7 Marks]

b) Show that the function $e^x(\cos y + i \sin y)$ is an analytic function, find its derivative. [5 Marks]

3a) Write all possible Laurent series for the function $f(z) = \dfrac{1}{z(z+2)^3}$ about the pole $z = -2$ [7 Marks]

b) Find $\dfrac{df}{dz}$ of this function: $f(z) = 4x + y + i(-x + 4y)$ along

(i) imaginary axis [3 Marks]

(ii) a line when $y = x$ [2 Marks]

4. a) Find the image of the circle $|z-1|=1$ in the complex plane under the mapping $w=\frac{1}{z}$. [7 Marks]

b) Evaluate $\int_{3i}^{2+4i}(2y+x^2)\,dx+(3x-y)\,dy$ along the parabola $x=2t$ and $y=t^2+3$. [5 Marks]

5a) Evaluate the following integrand using Cauchy integral formula $\int_c \frac{4-3z}{z(z-1)(z-2)}\,dz$ where

c is the circle $|z|=\frac{3}{2}$. [8 Marks]

b) Find the region of convergence of the series $\sum_{n=1}^{\infty} n!\,z^n$. [4 Marks]

6a) Find the smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n=1$ [5 Marks]

. b) If $z_1=2+i$ and $z_2=3-2i$, show that $\left|\frac{2z_2+z_1-5-i}{2z_1-z_2+3-i}\right|^2=1$. [7 Marks]