

NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.
FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
September 2020_1 Examination
Course Code: MTH 303
Course Title: Vectors and Tensors Analysis
Credit Unit: 3
Time Allowed: 3 Hours

## Instruction: Answer Question Number One and Any Other Four Questions

1. (a) (i) Show that the angle between two vectors $\underline{a}=\left(a_{1}, a_{2}, a_{3}\right)$ and $\underline{b}=\left(b_{1}, b_{2}, b_{3}\right)$ can be expressed as $\theta=\cos ^{-1} \frac{a_{1} b_{1}+a_{2} b_{2}+a_{3} b_{3}}{|\underline{a}||\underline{b}|} \quad$ (4 marks)
(ii) Give the definition of the magnitude of a vector product in terms of the area of a parallelogram.
(iii) Show that if $\underline{a} \wedge \underline{b}=0$ and neither $\underline{a}=0$ nor $\underline{b}=0$, then $\underline{a}$ and $\underline{b}$ are parallel.
(b) Define the followings:
(i) Derivative of a vector function $\underline{A}(u)$
(ii) Gradient of the function $\emptyset(x, y, z)$
(2 marks)
(iii) the curl of a vector function $\underline{A}$
(3 marks)
(c) (i)State the Divergence Theorem
(3 marks)
(ii) State Stoke's Theorem
(3 marks)
2. (a) If $\underline{A}, \underline{B}, \underline{C}$ are differentiable vector functions of a scalar u , give the expressions for the following derivatives. I. $\frac{d}{d u}(\underline{A} \cdot(\underline{B} \wedge \underline{C}))$ II. $\frac{d}{d u}(\underline{A} \wedge(\underline{B} \wedge \underline{C}))$
(b) Given a vector $\underline{Q}=\cos 3 t i+\sin 3 t j$, obtain $\left|\frac{d \underline{Q}}{d t}\right|$
(c) Show that Grad $\emptyset \cdot d r=d \emptyset$
3. (a) Show that $\frac{d \emptyset}{d s}=\frac{d \underline{r}}{d s} \cdot \operatorname{Grad} \emptyset$
(4 marks)
(b) If $\emptyset(n, y, z)=2 n^{2} y z^{2}$, find $\nabla \emptyset$
(4 marks)
(c) If $\emptyset=n^{2} y z$ a scalar function and $\underline{A}=2 n z i+y z j-n y^{2} k$, find $\nabla \cdot(\varnothing \underline{A})$ at the point (1, -1, 1).
4. (a) What is summation convention?
(b) Define the contravariant component of a tensor of the second rank.
(c) Explain the terms 'Outer product of tensors' and 'Contraction'
(6 marks)
5. (a) Evaluate $\int_{c}(x+3 y) d x$ from $\mathrm{A}(0,1)$ to $\mathrm{B}(2,5)$ along the curve $\mathrm{y}=1+\mathrm{x}^{2}$. ( 6 marks)
(b) Evaluate $I=\int_{c}\left\{\left(x^{2}+2 y\right) d x+x y d y\right\}$ from $\mathrm{O}(0,0)$ to $\mathrm{B}(1,4)$ along the curve $\mathrm{y}=$ $4 x^{2}$.
(6 marks)
6. (a) Evaluate the double integral $\iint_{R}\left(x^{2}-y^{3}\right) d x d y, 1 \leq x \leq 2,2 \leq y \leq 4$ (6 marks)
(b) A solid is enclosed by the planes $\mathrm{z}=0, \mathrm{y}=1, \mathrm{y}=2, \mathrm{x}=0, \mathrm{x}=3$ and the surface $\mathrm{z}=\mathrm{x}$
$+y^{2}$. Determine the volume of the solid so formed
