



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
**Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.**

**FACULTY OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**  
**SEPTEMBER Examination 2020\_1**

**Course Code: MTH 302**

**Course Title: Elementary Differential Equations**

**Credit Unit: 3**

**Time Allowed: 3 Hours**

**Instruction: Answer Question Number One and Any Other Four Questions**

1. (a) i. Define Power Series (2 marks)

ii. When is a Power Series said to be convergent? (2 marks)

(b) i. Define Radius of Convergence (2 marks)

ii. Determine the Radius of Convergence of the following Power Series

i.  $f(x) = \sum_{n=0}^{\infty} \frac{2^n}{2^{n+1}}$  (2 marks)

ii.  $f(x) = \sum_{n=0}^{\infty} \frac{2^n(n+1)^2}{n^2 2^{n+1}}$  (2 marks)

(c). (i) Define Orthogonality with respect to the weight function (2 marks)

(ii) When is a function said to be even? Give an example (2 marks)

(iii) When is a function said to be odd? Give an example (2 marks)

(d) If  $P(x) \frac{d^2 y}{dx^2} + Q(x) \frac{dy}{dx} + R(x)y = 0$

Where  $P(x)$ ,  $Q(x)$ , and  $R(x)$  are polynomials.

i. Define the ordinary point of equation (1) (2 marks)

ii. Define the singular point of equation (1) (2 marks)

iii. Define the regular singular point of equation (1) (2 marks)

2. Given the equation  $(1 - x^2)y'' - 6xy' - 4y = 0$ , near the ordinary point  $x = 0$ .

(i) Show that the solution is invalid in  $|x| < 1$ . (5 marks)

(ii) Obtain the recurrence relation for the equation and hence solve for y (7 marks)

3. Solve the equation  $y'' + (x - 1)^2 y' - 4(x - 1)y = 0$  **(12 marks)**

4. (i) State the general form of a linear homogeneous second order boundary value problem

(ii) Given that  $y'' + \gamma y = 0, y(0) = 0, y'(0) = 0$  **(3 marks)**

Show that if  $\phi_m$  and  $\phi_n$  are Eigen functions corresponding to the Eigen values  $\gamma_m$  and  $\gamma_n$  respectively, then  $\int_0^1 \phi_m(x)\phi_n(x)dx = 0$ , provided  $\gamma_m \neq \gamma_n$  **(9 marks)**

5. (i) Solve the equation  $x^2 y'' + \alpha x y' + \beta y = 0$

With the transform  $x = e^z$  or  $z = \log x$ , and  $x > 0$ . **(6 marks)**

(ii) Describe the solution for different type of roots. **(6 marks)**

6. (i) Obtain the Fourier series over the interval  $0 \leq x \leq 2$  for the function  $f(x) = 2, x = 2$  **(6 marks)**

(ii) Prove that all the Eigen values of the Sturm-Liouville problem

$$L(y) = \gamma r(x)y$$

with boundary condition

$a_1 y(0) + a_2 y'(0) = 0, b_1 y(I) + b_2 y'(I) = 0$ , are real **(6 marks)**