NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2020_2 EXAMINATION 1234

COURSE CODE: MTH301
COURSE TITLE: Functional Analyis
Time Allowed: 3 Hours
Total Marks: 70 Marks
Instruction: Answer Question One and Any other Four questions

(1a) (i) Define an ordered field. **(6 Marks)**

(ii) When does a pair (X, d), which contains a nonempty set X and a real- valued function d , said to be pseudometric? **(2 Marks)**

(b) (i) What is an open ball? **(2 Marks)**

(ii) Let $x \in \Re^n$. Prove that the set $B(x, \varepsilon)$, is open for some $\varepsilon > 0$. **(4 Marks)**

(iii) Define a closed set in $\Re^n$. **(2 Marks)**

(c) Prove that in $\Re^n$,

(i) the union of arbitrary collection of open sets is open. **(3 Marks)**
(ii) the finite intersection of a collection of open sets is open. **(3 Marks)**

(2a) (i) When is a set E said to be meager or of first Baire category? **(2 Marks)**

(ii) Let X be a complete metric space and $\{O_n\}$ be a countable collection of dense open subsets of X. Prove that $\bigcup O_n$ is not empty**.** **(4 Marks)**

(b)(i) Define a countable set. **(2 Marks)**

(ii) What is the closure of a subset S of a metric space. **(2 Marks)**

(iii) Prove that in $\Re^n$, every family of disjoint nonempty open set is countable **(2Marks)**

(3a) (i) Define a continuous function of metric spaces. **(2 Marks)**

(ii) Let A and B be two metric spaces. Prove that the function $f: A \to B$ is continuous if and only if $f^{-1}(V)$ is an open set in A, where V is an open set In B **(4 Marks)**

(b)(i) Define convergent sequence in a metric space. **(2 Marks)**

(ii) Let (X,d) be a metric space. prove that a subset A of X is closed in (X, d) if and only if every convergent sequence of points in A converges to a point in A. **(4 Marks)**

(4a) (i) Define a function from $\Re^N$ *to* $\Re^M$. **(2 Marks)**

(ii) Let (X, d) and (Y,d_1) be metric spaces and f a mapping of X into Y. Let τ *and* τ_1 be the topologies defined by d and $d_1$respectively. Prove that $f:(X,\tau) \to (Y,\tau_1)$ is continuous if and only if $x_n \to x \Rightarrow f(x_n) \to f(x)$, that is, if $x_1, x_2, \ldots x_n$ is a sequence of points in (X, d) converging to x, then the sequence of points $f(x_1), f(x_{2,}), \ldots, f(x_n)$ in (Y, d_1) converging to $f(x_n)$. **(4 Marks)**

(b)(i) Define an open ball around a point x in a metric space S. **(2 Marks)**

(ii) Explain the concepts of open, closed and infinite intervals on the real line. **(4 Marks)**

(5a)(i) Define the neighbourhood of a point in a metric space. **(2 Marks).**

(ii) Let (K, d_k) be a compact metric space. Let (Y, d_Y) be any metric space and let $f: K \to Y$ be a continuous function. Prove that f(K) is compact. **(3 Marks)**

(5b)(i) Define a connected topological space. **(2 Marks)**

(ii) Let (K, d) be a compact metric space. prove that every sequence in K has a convergent subsequence**.** **(5 Marks)**

(6a) Define

(i) A compact subset of a metric space. **(2 Marks)**

(ii) Open cover of a subset of a metric space. **(2 Marks)**

(iii) A totally bounded metric space. **(2 Marks)**

(b) Let X be a metric space and let Y be a subspace of X. Prove that

(i) If X is compact and Y is closed in X, then Y is compact. **(3 Marks)**

(ii) If Y is compact, then Y is closed in X. **(3 Marks)**