NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
2021 Examinations

Course Code: STT 311
Course Title: Probability Distributions II
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 4 Questions

Question 1 (Compulsory)

(a) State clearly the Chebyshev's Inequality **(4 marks)**

(b) A sample of 3 balls taken from an urn is examined to determine whether the colour is red (R) or blue (B).

i. Determine the appropriate sample space for this experiment **(2 marks)**

Hence, state elements of the

ii. Event E_1 that exactly one of the balls picked is blue **(2 marks)**

iii. Event E_2 that the first ball picked is blue **(2 marks)**

iv. Event E_3 that there are exactly two red balls in the sample **(2 marks)**

(c) Given that **x** is a Bernoulli random variable, state the probability mass function of **x** with probability of success as **p** and probability of failure as **(1 – p)** **(2 marks)**

(d) Given the following moment generating functions (MGFs), determine the distribution of the random variable ***x*** having the MGFs. Hence or otherwise, find the mean of ***x***.

i. $M_x(t) = e^{8t+5t^2}$ **(2 marks)**

ii. $M_x(t) = \frac{13}{13-t}$ **(2 marks)**

iii. $M_x(t) = (1 - 5t)^{-6}$ **(2 marks)**

iv. $M_x(t) = (0.75 + 0.25e^t)^{48}$ **(2 marks)**

Question 2

A semi-boarding school in Abuja has 30% female and 70% male students. The record shows that 20% of the female students and 40% of the male students are boarders while the rest are day students. If a boarding student is picked at random, determine the probability that the student is a male using your idea of Bayes' theorem. **(12 marks)**

Question 3

A box of compact fluorescent lamps (CFLs) has 10 lamps of which 4 are defective due to vibration during vehicular transit from the factory. If two lamps are selected at random without replacement, find the probability that:

i. Both lamps are defective **(3 marks)**

ii. Both lamps are non-defective **(3 marks)**

iii. Only one lamp is defective **(3 marks)**

iv. At least one lamp is defective **(3 marks)**

Question 4

An experiment involves planting 3 seedlings in a yard. Each of the seedings has independent chance to survive (**S**) or not survive (**N**) while the probability that each seedling will survive is 0.75. Let a random variable $\boldsymbol{x}$ be the number of surviving trees after one year of planting.

(a) Construct a probability distribution for this experiment **(6 marks)**

(b) Hence, determine the expected value of the random variable x **(6 marks)**

Question 5

The joint density function of the continuous random variables **x** and **y** is given by

$$f(x,y) = \begin{cases} c\,x\,y, 0 \le x \le 1, 0 \le y \le 1 \\ 0, \quad otherwise \end{cases}$$, determine

(a) the value of c **(4 marks)**

(b) the marginal density function of **x** **(4 marks)**

(c) the marginal density function of **y** **(4 marks)**

Question 6

The joint probability distribution of the number of taxi cabs and the number of buses that arrive the University terminus park per one-hour interval on a particular Monday morning is given below.

	Number of Taxi Cabs				
	0	1	2	3	Total
Number of Buses 0	4λ	0.09	0.06	θ	β
1	0.08	0.03	0.14	0.12	0.37
2	θ	0.13	0.10	λ	0.31
Total	α	0.25	0.30	0.20	

Determine the value of (i) α **(3 marks)**

(ii) β **(3 marks)**

(iii) θ **(3 marks)**

(iv) λ **(3 marks)**