NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OFMATHEMATICS
Examination 2021

Course Code:	**MTH 422**
Course Title:	**PDE**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question Number one and any Other Four Questions**

1. (a). Classify the following according to type

i. $u_{xx} + 2yu_{xy} + xu_{yy} - u_x + u = 0$

ii. $2xyu_{xy} + xu_y + yu_x = 0$

iii. $u_{xx} + u_{xy} + 5u_{yx} + u_{yy} + 2u_{yz} + u_{zz} = 0$ **(2 marks each)**

iv. $2u_{xx} - 4u_{xy} - 6u_{yy} + u_x = 0$

v. $4u_{xx} + 12u_{xy} + 9u_{yy} - 2u_x + u = 0$

(b). Solve the following PDE for *u(x ,t)* using the methods of characteristics

$\frac{\partial u}{\partial t} + 5\frac{\partial u}{\partial x} = 0, \quad -\infty < x < \infty, t > 0$

$and\ u(x,0) = e^{-x^2}$ **(4 marks)**

(c). Suppose that $u = u(x,t)$ and $v = v(x,t)$ have partial derivatives related in the following way:

$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial t} = -\frac{\partial u}{\partial x}$

Show that u and v are solution of the wave equation $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$ with $c = 1$
(4 marks)

(d). Use the change of variables $\alpha = x + ct$, $\beta = x - ct$ to transform the wave equation $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$ into $\frac{\partial^2 u}{\partial \alpha \partial_\beta} = 0$. (you should assume that $\frac{\partial^2 u}{\partial \alpha \partial_\beta} = \frac{\partial^2 u}{\partial_\beta \partial \alpha}$.)
(4 marks)

2. (a). Using the following solution $u(x,t) = F(x + ct) + G(x - ct)$ solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

for the data $u(x,0) = e^{-x^2}, \frac{\partial u}{\partial t}(x,0) = 0, -\infty < x < \infty.$ **(6 marks)**

(b). Solve $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, $u(0,t) = 0, for\ t \geq 0\ and\ u(L,t) = 0, for\ t \geq 0$ and initial conditions , $u(x,0) = f(x), for \quad 0 \leq x \leq L$

$and\ \frac{\partial u}{\partial t}(x,0) = g(x)\ for\ \ 0 \le x \le L$ for the data:

$f(x) = \frac{1}{2}\sin\frac{\pi x}{L} + \frac{1}{4}\sin\frac{3\pi x}{L} + \frac{2}{5}\sin\frac{7\pi x}{L}, \quad g(x) = 0$ **(6 marks)**

3. (a). Using the following solution $u(x,t) = F(x+ct) + G(x-ct)$ solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

for the data $u(x,0) = e^{-x^2}, \frac{\partial u}{\partial t}(x,0) = 0, -\infty < x < \infty.$ **(6 marks)**

(b). Solve $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}, \quad u(0,t) = 0, for\ t \ge 0\ and\ u(L,t) = 0, for\ t \ge 0$ and initial conditions , $u(x,0) = f(x), for \quad 0 \le x \le L$

$and\ \frac{\partial u}{\partial t}(x,0) = g(x)\ for\ \ 0 \le x \le L$ for the data:

$f(x) = \frac{1}{2}\sin\frac{\pi x}{L} + \frac{1}{4}\sin\frac{3\pi x}{L} + \frac{2}{5}\sin\frac{7\pi x}{L}, \quad g(x) = 0$ **(6 marks)**

4. a. Using the solution $u(x,t) = F(x+ct) + G(x-ct)$ of the wave equation $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$

where F and G are arbitrary differentiable functions of one variable, solve the wave equation with initial data $u(x,0) = \frac{1}{1+x^2}, \quad \frac{\partial u}{\partial t}(x,0) = 0, \quad -\infty < x < \infty,$ **(6 marks)**

b. Solve the wave equation with initial data

$u(x,0) = \frac{x}{1+x^2}, \quad \frac{\partial u}{\partial t}(x,0) = -2xe^{-x^2}$ **(6 marks)**

5. (a). Find the dimension and order of the following PDEs. Which are linear, and which are homogeneous ?

i. Heat equation: $u_t = Du_{xx} + f(x)$ **(2 marks)**

ii. Wave equation: $u_{tt} - c^2u_{xx} = 0$ **(2 marks)**

iii. Laplace's equation: $u_{xx} + u_{yy} = 0$ **(2 marks)**

iv. Kdv equation: $u_t + uu_{xx} + u_{xxx} = 1$ **(2 marks)**

(b). Solve the following PDE for $u(x,t)$ using the method of characteristics

$L = \frac{\partial u}{\partial t} + 5\frac{\partial u}{\partial x} = 0, \quad -\infty < x < \infty, \quad t > 0, u(x,0) = e^{-x^2}$ **(4 marks)**

6. (a). Suppose that u_1 and u_2 are solutions of $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$, show that $c_1u_1 + c_2u_2$ is also a solution, where c_1 and c_2 are constants. **(2 marks)**

(b). Solve the normalized wave equation

$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \qquad 0 < x < \pi, \ ,t > 0$

$u(0,t) = 0, \quad u(\pi,t) = 0$

$u(x,0) = \sin x, \quad \frac{\partial u}{\partial x}(x,0) = \sin x$ **(10 marks)**