NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_1 Examinations

Course Code: **MTH 411**
Course Title: **Measure Theory and Integration**
Credit Unit: **3**
Time Allowed: **3 Hours**
Instruction: **Attempt Number One (1) and any four (4) Questions**

1. (a) Define the measure of a bounded open set. **(3 marks)**
(b) Define the measure of a non – empty bounded closed set F. **(3 marks)**
(c) State Minkowski inequality. **(3 marks)**
(d) Let (X, fl) be a measure space. Let $f_n: X \to \mathbb{R}$ be a sequence of measurable functions converging pointwise to f. Moreover, suppose that there is an integrable function g such that $|f_n| \le g$ for all n. Show that f_n and f are also integrable and $\lim_{n\to\infty} \int_X |f_n - f|$dfl = 0.
(7 marks)
(e) Let (X, $\mathcal{M}$) be a measurable space. Explain when a set function fl whose domain is the q− algebra $\mathcal{M}$ is called
(i) additive and **(3 marks)**
(ii) countably additive. **(3 marks)**

2. (a) Define counting measure on (X, M), which is a measurable space. **(4 marks)**
(b) Distinguish between measurable function and Borel function with four examples. **(8 marks)**

3. (a)) Let (X, M, fl) be a measure space, and let f and g be extended real-valued functions on X that are equal almost everywhere. If fl is complete and if f is measurable, explain that g is measurable. **(6 marks)**
(b)) Let G_1 , G_2 be open sets such that $G_1 \subseteq G_2$, prove that m(G_1) $\le$ m(G_2). **(6 marks)**

4. (a) When is S: X → ℝ a simple function? **(2 marks)**
(b) Let { n } be a sequence of measurable functions. n : X → C a.e. Suppose that

$\Sigma_{n=1} \int_x |\ n\ |\ d$ fl $< \infty$. Show that $\Sigma_{n=1}\ {}_n(x)$ converges to f(x) a.e on X. **(10 marks)**

5. (a) State Beppo Levi's theorem. **(4 marks)**

   (b) Let (X, M, fl) be a measure space and let A and B be subsets of X that belong to M and satisfy A $\subseteq B$. Show that fl (A) $\leq$ fl (B). If in addition A satisfies fl(A) < +∞, then fl(B – A) = fl(B) – fl(A). **(8 marks)**

6. (a) Let $f_n: X \to [0, \infty]$ be non – negative measurable functions. Show that $\int \sum_{n=1}^{\infty} f_n = \sum_{n=1}^{\infty} \int f_n$. **(6 marks)**

   (b) Suppose that fl is Lebesque measure and that f is defined as follows:
   f(x) = {$4\ if -3 < x < 3; 5\ if\ 3 \leq x < 7; 8\ if\ 7 \leq x < 9; 1\ if -7 < x \leq -3;$
   2 if -9< x ≤ -7; 0 otherwise. Find $\int f(r)$fl(dr). **(6 marks)**