NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_1 Examinations

Course Code: MTH402
Course Title: General Topology II
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other Four (4) Questions

1. (a) Explain discrete and indiscrete topologies. **(6 marks)**
   (b) Let X be a set. Let a topology on X be a collection τ of subsets of X. Show that finite intersections $\bigcap_{k=1}^{n} U_k$ of elements of τ are in τ. **(8 marks)**
   (c) Let X be a set, and let B be a basis for a topology τ on X. Then show that τ equals the collection of all unions of elements of B. **(8 marks)**
2. (a) Let d be a metric on the set X. Show that the collection of all r - balls $B_d(x, r)$, for $x \in X$ and $r > 0$ is a basis for a topology on X, called the metric topology induced by d. **(5 marks)**
   (b) Prove that $x \in \bar{A}$ if and only if every neighbourhood of x intersects A. i.e., $x \in \bar{A}$ if and only if for all $V \in N(x)$, $V \cap A \neq \emptyset$, where A is a subset of a topological space X. **(7 marks)**
3. (a) Let **B** and $\mathbf{B}^0$ be basis for the topologies τ and τ^0 respectively on X. Show that the following are equivalent:
   i. τ^0 is finer than τ.
   ii. For each $x \in X$ and each element $B \in \mathbf{B}$ containing x, there exists a basis element $B^0 \in \mathbf{B}^0$ such that $x \in B^0 \subset \mathbf{B}$. **(6 marks)**

   (b) State whether each of the following functions is a homeomorphism or not:
   (i) f: R →R given by f(x) = 4x + 1. **(2 marks)**
   (ii) F: (-1, 1) →R given by $F(x) = \dfrac{x}{1-x^2}$. **(2 marks)**
   (iii) The identity map g: $\mathbb{R}_1 \to \mathbb{R}$ is bijective and continuous. **(2 marks)**

4. (a) What is the relationship between T_1 – space, T_3 – space and regularity defined on a topological space X? **(3 marks)**
(b) Show that every Hausdorff space is T_1 and the converse is not true. **(4 marks)**
(c) Show that Q, the set of rational numbers is a dense subset of R because $\bar{Q} = R$. **(5 marks)**

5. (a) State whether each of the following is countable or not countable:
(i) Z **(1 mark)**
(ii) The image of a countable set under any map. **(1 mark)**
(iii) R. **(1 mark)**
(iv) The set $N^2 = \{(k, n) : k, n \in N\}$. **(1 mark)**
(v) The union of a countable family of countable sets. **(1 mark)**
(vi) Q. (1 mark)
(b)) Show that f is a homeomorphism if X is compact, Y is Hausdorff and $f : X \to Y$ is a continuous bijective function. **(6 marks)**

6. (a) Define the following terms:
(i) Covering and Open Cover **(2 marks)**
(ii) Compact Set **(2 marks)**
(iii) Subcover **(2 marks)**
(b) Show that h is continuous if h: R → R is defined by

$$h(x) = \begin{cases} = \frac{x}{2}, if\ x\ \geq 0 \\ = \text{x, if } x \leq 0 \end{cases}$$ **(6 marks)**.