NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_1 Examinations

Course Code: **MTH401**
Course Title: **General Topology**
Credit Unit: **3**
Time Allowed: **3 Hours**
Total: **70 Marks**
Instruction: Answer Question One (1) and Any Other 4 Questions

1a) State without prove the Holder's inequality. **(4 marks)**

b) If $f: \mathbb{R}^n \longrightarrow \mathbb{R}$, when is f said to be continuous at $a = (a_1 \cdots a_n)$. **(4 marks)**

c) Show that if (E, d) is a metric space, $a \in E$(a fixed element) and $f: E \longrightarrow \mathbb{R}$ such that $f(x) = d(x, a)$ for all $x \in E$. Then f is uniformly continuous on E. **(6 marks)**

d) Given that $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ such that

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & \text{for } x^2+y^2 \neq 0 \\ 0, & \text{for } x = y = 0 \end{cases}$$

Verify the continuity of f at $(0,0)$. **(8 marks)**

2a) Define a metric on a nonempty set E. **(3 marks)**

b) Verify that the usual metric on $\mathbb{R}$ is a metric **(5 marks)**

c) State without prove the Cauchy schwartz's inequality. **(4 marks)**

3a) Define the following terms:

(i) an open set in a metric space **(1 marks)**
(ii) an interior point in a metric space **(1 marks)**
(iii) a closed set in a metric space. **(1 marks)**

b) Let $E = \mathbb{R}^2$ be endowed with the Euclidean metric$d_2(x,y)=(\sum_{k=1}^{n}|x_k - y_k|^2)^{\frac{1}{2}}$ for all $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$. Describe the sets:

i) $B\big((0,0);1\big)$ **(2 marks)**

ii). $\overline{B}\big((0,0);1\big)$ **(2 marks)**

iii) $S\big((0,0);1\big)$ where $(0,0) \in \mathbb{R}^2$n **(2 marks)**

iv). $B_r(x_0, y_0)$ for arbitrary where $x_0, y_0 \in \mathbb{R}^2$ and $r > 0$ **(3 marks)**

4a) Define the following terms

i) Cauchy Sequence' **(2 marks)**
ii) Bounded sequence **(2 marks)**

iii) Cluster point **(2 marks)**

b) Let (E, d), be an arbitrary metric space and let $\{x_n\}$ be a Cauchy sequence in E. Then is bounded **(6 marks)**

5a) State without prove the Pasting Lemma on union of closed sets **(4 marks)**

bi) Given that $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ such that

$$f(x,y) = \begin{cases} x^2 \mathrm{Sin}\frac{1}{y} + y^2 \mathrm{Sin}\frac{1}{x}, & \text{for } x \neq 0, y \neq 0 \\ 0, & \text{for } x = 0, y = 0 \end{cases}$$

Verify the continuity of f at $(0,0)$ **(4 marks)**

bii) Given that $f: \mathbb{R} \longrightarrow \mathbb{R}$ such that

$$f(x) = \begin{cases} x^2 + 1, & \text{for } x \leq 0 \\ \frac{1}{2}(x+2), & \text{for } x \geq 0 \end{cases}$$

Verify the continuity of f on $\mathbb{R}$. **(4 marks)**

6ai) what is a connected metric space? **(2 marks)**

ii) What is a connected subspace of a metric space **(2 marks)**

bi)Show that $K = (0,2) \cap [3,4]$ is a subspace of a metric space (E, d) is disconnected. **(4 marks)**

bii)Show that $\mathbb{R}^2$ is connected. **(4 marks)**