NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMATICS
2021_1 Examinations

Course Code: MTH312
Course Title: Abstract Algebra
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 4 Questions

1a) Define the following terms:
(i) Normal subgroup of a group G **(2 Marks)**
(ii) Commutator of x and y, $x, y \in G$ **(2 Marks)**
(iii) a ring with identity iv) an even permutation. **(2 Marks)**

b) Show that if $f: G_1 \longrightarrow G_2$ is a homomorphism, then
(i) $\text{Ker } f$ is a normal subgroup of G_1. **(3 1/2 Marks)**
(ii) $\text{Im } f$ is a subgroup of G_2 **(3 1/2 Marks)**

c)Let R be a ring and $a \in R$. Show that the set $aR = \{ax : a \in R\}$ is a subring of R. **(7 Marks)**

2a) If R_1 and R_2 are two rings and $f: R_1 \longrightarrow R_2$ is a ring homomorphism. Define the followings

(i) $\text{Im } f$ **(1 Mark)**
(ii) $\text{Ker } f$ **(1 Mark)**
(iii) Ring isomorphism. **(1 Mark)**

bi) Given that $f: R_1 \longrightarrow R_2$ is a ring homomorphism, f is surjective and I is an ideal of R_1. Show that $f(I)$ is an ideal of the ring R_2. **(4.5 Marks)**

bii) If I is an ideal of a ring R, show that there exists a ring homomorphism $f: R \longrightarrow R/I$ whose kernel is I. **(4.5 Marks)**

3a) Define the following terms

(i) ideal of a ring **(2 marks)**

(ii) proper ideal of a ring **(2 marks)**

(iii) The ideal generated by $a_1, a_2, \cdots, a_n$, elements of a ring. **(2 marks)**

bi) Given that X is an infinite set and I is the class of all finite subsets of X. Show that I is an ideal of $\wp(X)$. **(3 Marks)**

bii) For any ring R and $a_1, a_2 \in R$. Show that $Ra_1 + Ra_2 = \{x_1a_1 + x_2a_2 \in R\}$ is an ideal of R.

(3 Marks)

4a) Explain the following terms i. when a permutation is called r-cyclic. ii. A transposition iii. When two cycles are said to be disjoint iv. The signature of $f \in S_n$. **(6 Marks)**

b) Express each of the following permutations as products of disjoint cycles.

i. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 2 & 1 & 3 \end{pmatrix}$ ii. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 3 & 1 & 2 \end{pmatrix}$

iii. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 4 & 7 & 2 & 1 & 3 & 6 & 5 \end{pmatrix}$

(3 Marks)

c) Given that $f, g \in S_n$, show that $\text{sign}(f^\circ g) = (\text{sign } f)(\text{sign } g)$ **(3 Marks)**

5a) Show that $\text{Aut}\mathbb{Z} \cong \mathbb{Z}_2$ **(6 Marks)**

b) Show that any cyclic group is isomorphic to $(\mathbb{Z}, +)$ or $(\mathbb{Z}_n, +)$. **(6 Marks)**

6a) Define the following terms

(i) Principal ideal **(2 Marks)**

(ii) Nilpotent **(2 Marks)**

(iii) Nil radical of R. **(2 Marks)**

b) Given a ring R and an ideal I. Show that R/I is a ring with respect to addition and multiplication defined by $(x + I) + (y + I) = (x + y) + I$ and $(x + I)(y + I) = (xy) + I$ for all $x, y \in R$. **(6 Marks)**