

NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja
FACULTY OF SCIENCES
Department of Mathematics
2021 Examinations

Course Code: MTH307
Course Title: Numerical Analysis II
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 4 Questions

1. (a) Given the general second order Partial Differential Equation

$$
L(u)=A u_{x x}+B u_{x y}+C u_{y y}-H\left(x, y, u, u_{x}, u_{y}\right)=0
$$

Classify the following equation into Parabolic, Elliptic and Hyperbolic.
i. $\quad u_{t}=u_{x x}$
ii. $\quad u_{t t}=u_{x x}$
iii. $\quad u_{x x}+u_{y y}=0$
(5 marks)
(b) When is Partial Differential Equation said to be Parabolic, Elliptic and Hyperbolic.
(5 marks)
(c) Solve the Laplace equation $u_{x x}+u_{y y}=0$, subject to the boundary conditions

$$
\begin{equation*}
u(x, 0)=1, u(0, y)=0, u(1, y)=0, u(x, 1)=1 ; 0 \leq x \leq 1,0 \leq y \leq 1 . \tag{12marks}
\end{equation*}
$$

2. (a) Express the function $x^{3}+2 x^{2}-x-3$ in terms of Legendre polynomials.( 5 marks)
(b) Find the fourth degree least square polynomials to $|x|$ over $[-1,1]$ by means of Legendre polynomials.
(7 marks)
3. (a) Show that $T_{n}(x)$ satisfies the differential equation $\left(1-x^{2}\right) y^{\prime \prime}-x y^{\prime}+n^{2} y=0$
(5 marks)
(b) Convert the first 5 terms of the Taylor series expansion for $e^{x}$ into Chebyshev
polynomials.
4. (a) Determine the parameters in the formula
$p(x)=a_{o}(x-a)^{3}+a_{1}(x-a)^{2}+a_{2}(x-a)+a_{3}$ such that
$p(a)=f(a), p^{\prime}(a)=f^{\prime}(a), p(b)=f(b), p^{\prime}(b)=f^{\prime}(b)$.
(5 marks)
(b) Obtain the unique polynomial $p(x)$ of degree 5 or less approximating the function $f(x)$, where $f\left(x_{0}\right)=1, f^{\prime}\left(x_{0}\right)=2$. Also, find $p\left(\frac{\left(x_{0}+x_{1}\right)}{2}\right)$.
5. (a) Distinguish between Trapezoidal and Simpson's rule.
(b) Evaluate $\int_{1}^{3} \frac{1}{x+1} d x$ using the Simpson's one-third rule with $h=\frac{1}{4}$, working with four floating point arithmetic.
6. (a) State the properties of a cubic spline interpolation.
(b) Obtain the cubic spline from data below and compute $y(1.5)$ and $y^{\prime}(1)$.

| $X$ | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- |
| $Y$ | -8 | -1 | 18 |

