NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_1 Examinations

Course Code: MTH 305
Course Title: Complex Analysis II
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 4 Questions

Q1 (a) Define each of the following:

(i) Limit of a complex function $f(z)$. **(4 marks)**
(ii) Essential singularity. **(2 marks)**

(b) Establish that $sin^2 z + cos^2 z = 1$ **(6 marks)**

(c) Determine the poles and the residues at the poles of $f(z) = \frac{3z+1}{(z^2-z-2)}$ **(6 marks)**

(d) State the Residue theorem. **(4 marks)**

Q2 (a) State the Cauchy integral formula **(3 marks)**

(b) If c is a curve $y = x^3 - 3x^2 + 4x - 1$ joining the points (1,1) and (2,3),

show that $\int_c (12z^2 - 4iz)dz$ is independent of the path joining (1,1) and (2,3).
(9 marks)

Q3 (a) Differentiate between a single valued and a multiple valued complex function $w(z)$.
(3 marks)

(b) Prove that $cosh^2 z - sinh^2 z = 1$ **(9 marks)**

Q4 (a) Define each of the following:

(i) A continuous complex function f at a point. **(3 marks)**

(ii) bounded complex function. **(2 marks)**

(b) Find the Laurent series expansion of $f(z) = \frac{1}{z-3}$ valid for $|z| < 3$. **(7 marks)**

Q5 (a) Define a harmonic function. **(4 marks)**

(b)The derivative of the function $f(z) = z^2$ exists everywhere,
Show that the Cauchy-Riemann equations are satisfied everywhere. **(8 marks)**

Q6 (a) Define an isolated singular point. **(3 marks)**

(b) Determine the poles and the residues at the poles of $f(z) = \left(\frac{z+1}{z-1}\right)^2$. **(7 marks)**

(c) State the Morera's theorem **(2 marks)**