NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_1 Examinations

Course Code:	**MTH304**
Course Title:	**COMPLEX ANALYSIS**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question One (1) and Any Other 4 Questions**

1. a. Define a complex number. **(6 marks)**

   b. . Let c be the circle |z|=4. Evaluate the integral $\int_c \frac{\cos z}{z^2 - 6z + 5} dz$ **(10 marks)**

   c. What is the value of x, y in equations below:

       i. $(u, v) + (a, v) = (x, y)$ **(3 marks)**

       ii. $(8,1) + (x, y) = (10,1)$ **(3 marks)**

2. a. Prove that $|z + w|^2 \le |z|^2 + |w|^2$ **(4 marks)**

   b. Write in polar form $re^{i\theta}$ i. $-2\sqrt{3} + 3i, 1 + i$ **(4 marks)**

   c. What is a vector valued function? **(4 marks)**

3. a. Suppose $f(z) = z^2$. What is $\lim_{z \to 0} \frac{f(z) - f(z_0)}{z - z_0}$? **(4 marks)**

   b. Suppose the function f *given by* $f(z) = u(x, y) + iv(x, y)$
   has a derivative at $z = z_0 = (x_0, y_0)$. Derive Cauchy Riemann Equations. **(4 marks)**

   c. if f has a derivative at a point z, then its real and imaginary parts satisfied which equations(list them) **(4 marks)**

4. a. If $z = x + iy$, find the equation of the locus $\left|\frac{z+1}{z-1}\right| = 2$ **(4 marks)**

   b. When is a function f(z) said to be analytic? **(4 marks)**

   c. Define Cauchy integral formula **(4 marks)**

5 a. Define power series. **(4 marks)**

   b.   i. What are conjugates? **(2 marks)**

       Determine the values of x and y from the following

       ii. $(u, v) + (a, v) = (x, y)$ iii. $(8,1) + (x, y) = (10,1)$ **(2 marks)**

   c.   Show that $\frac{1}{z(z-1)} = \sum_{j=2}^{\infty} z^{-j}$ **(4 marks)**

6a. When are two complex n umbers said to be equal? **(4 marks)**

b. Find the $\lim_{x \to 0} \left\{ \frac{\tan x - x}{x^3} \right\}$ **(4 marks)**

c. Compute $\int_C \frac{1}{(z^2+4)^2} dz$ over the contour shown below **(4 marks)**